NOT SO SIMPLE MATH

Gordon Scott, March, 1998.

Contents:

Who might find this booklet useful?
Experienced and Beginning Teachers, Teachers of Teachers, Parents.

All may wish to have a new way of looking at early arithmetic or math, something they have not found in the textbooks they have been expected to use, or they may be just looking for some ideas to supplement the texts they are happy with.

In this booklet I will try to point out some places where children might have trouble with number work. Adults, including myself, can have difficulty with this. A task can be done for so long it is seen as a single, simple unit, when it really takes a series of little steps for most children. "547 + 364 = ___ " is an example.

In my Simple Math booklet I mainly talked about very basic number work that children in the early years of school would be faced with. The emphasis was on understanding and being able to handle the simplest questions, such as "7 + 4 = ___", quickly and accurately.

The subjects of this booklet are areas young children may be expected to deal with that are more complex, and may deserve more attention. I hope to show you that in many cases these can, and should be, reduced to a series of simpler steps. Knowing that these are really just a lot of little questions done in a certain way can give children an important feeling of confidence.

The emphasis is not just on the little pieces, but also on how they are related in some logical order, an order that may or may not have some flexibility.

Please contact me by mail or e-mail if you have any questions, comments, or advice for improvements. **NEW ADDRESS AS OF NOV. 15, 2000 : 15057 27A Ave., Surrey, B.C., Canada, V4P 1P1** If you have any questions, suggestions, or comments, address an email to **@telus.net** but first put **m.games** before it. (This address is split to attempt to foil programs which gather addresses for sale to junk mailers.)

Part 1, Word Problems:

A short story containing a question that can be answered by working with numbers that are directly and indirectly given in the story can simply be called a problem.

The child's task is to get the correct number answer, and relate it back to the story.

EXAMPLE: A 7 year old child picks up 5 shiny pebbles on the way to school. On the way home he picks up some more. At home he shows his mother that he has 11 pebbles.
How many did he pick up on the way home?

Easy?? Not for most of the youngest children.

What follows is an attempt to make adults aware that solving word problems is not so simple for many young children, particularly when reading is still a difficult task and logical thought isn't well developed. This attempt isn't intended to meet university standards for thoroughness and accuracy; just to provoke thought on the process involved in finding an answer to what appears to adults to be a simple question.

Remember, these are my opinions. You will often see 'I' and 'my' used, and you can take that as a reminder that these are just the thoughts of one teacher.

If you have your own thoughts I'll be happy to have provoked some thinking.

Problem solving requires a number of steps, and the failure to handle any one of these may result in an inability to complete the problem, or an inaccurate answer.

1. First you have to read the words, and for most of the very youngest children this means read aloud. Many very young children must be able to read, in the sense of 'say', all of the words contained in the problem. They are unable or unwilling to try to really understand a short story when they cannot decipher some of the words.

2. Next you have to interpret the words. Some young children have difficulty understanding the main ideas of the story if there are words whose meaning is not clear to them. Those words stop them from putting the ideas together to form a story. Every time they run into them the story falls apart as they focus on word meaning instead of story meaning.

Some children could answer the problem without knowing what "pebbles" are, but others would lose sight of the overall picture when they read that word. They may not have the confidence to say that "pebbles" are obviously something you find, and get on with the rest of it.

3.Then there's the question. It has to be related back to the story. Most adults wouldn't notice that "pebbles" isn't used in the question, but this could puzzle some children.

4. Then they have to sort out the numbers that relate to the question asked. Some children want to skip directly to the next step, working with the numbers, without relating them to real ideas. They will grab all the numbers they see, regardless if they have anything to do with the question or not. They would want to use the 7 without thinking of it as counting years of age.

5. The numbers have to be fitted together in an incomplete number sentence that mirrors the original word story, and the question.
5 + ___ = 11
This may be a mental, or mental and written, process.

6. The number sentence must be solved, and sometimes this is not so easy when the answer does not pop into the mind. Many children have difficulty turning an additive situation into a subtractive one.

Some may even need permission to do this before attempting it. Remember, they are just starting all this, and they may lack confidence.

It may also depend to some extent on how they have been taught the basic number facts.

Some may have to say to themselves that they will pretend to start with the 11 and take away the part they know was found on the way to school. In other words they will go back to the word story ideas to do 11 - 5 = ____ .

7. When this work is done correctly, it has to be related to the word story. The number answer may be 6, but what is the 6 in terms of the word story?

In a sense, the child has to go into the word story world, find what's missing, take what is needed from the story into the world of numbers, and come back out again into the word story world with something that will complete the story.

I hope you can see that this is not so simple for many young children.

EXAMPLE: A 7 year old child picks up 5 shiny pebbles on the way to school. On the way home he picks up 4 more pebbles. At home he adds them to his collection and finds he now has 15. How many did he have before he went to school?

This one has two steps and can be done in more than one way.

Then there's the word problem that has numbers counting different important things. Here the child has to keep in mind what job each number is doing.
The numbers are small, but for many children this word problem would be confusing.

EXAMPLE: A 7 year old child picks up 5 shiny pebbles on the way to school. On the way home he picks up some more. At home he shows his older brother that he has 11 pebbles. His older brother says he will trade him 10 pennies for 10 of the pebbles. If they do trade, what will the younger brother have then?
This next one has two steps requiring number work, and requires a child to separate the number of pennies from the number of pebbles.

EXAMPLE: A 7 year old child picks up 5 shiny pebbles on the way to school. On the way home he picks up some more. At home he shows his brother that he has 11 pebbles.

His brother says he will give him 2 pennies for each of the pebbles he found on the way home. If the younger brother agrees to this, what will he have then?

Can children be taught problem solving strategies?

Most children will need help to become confident in their ability to break word problems into manageable, ordered steps. I hope you can see that the examples I have given here are much too difficult to start with. They were only intended as examples for adults.

Beginning problems must be very simple, with no numbers unrelated to the question.

If you look at the seven points above you can see that **the ability to read well is most important in doing this work independently.**

Some children will have to have the problems read to them, and will have to have someone available to reread sections, and to explain words.

When reading ability is not the chief difficulty, then there are some strategies that children can learn to help them through the problem to an accurate answer.

Most beginning problems are situations where people or things are being put together, or taken apart.

Children can practise recognizing each of these situations, without having to worry about working with numbers.

When children have been taught, as in my Simple Math booklet, to think in terms of two situations, **PART + PART = WHOLE** and **WHOLE - PART = PART** , they should find it easier to think, **"Are parts going together, or being taken apart?"**

Children can practise this with numbers that are impossible for most to work with. The example problem given first could have had 517 pebbles found on the way to school and 978 counted out at home.

The answer might not leap out at them, but the process remains the same, and the process is most important.

Children like to discover that some things do not change, that no matter what the numbers are, the problems remains essentially the same.
It gives them a very important feeling of confidence, and **being confident in one's ability is a very large factor in one's success in math.**

Children could practise a few simple problems over and over, using different numbers to reinforce the idea that problems can be very similar.

When they are competent at recognizing addition and subtraction situations, then they can practice pulling out the relevant numbers from the story and putting them into a number sentence that mirrors the story.

Again, this need not mean that the children focus on finding the answer. Finding the answer may take away from developing this separate skill.
Putting in large numbers that they cannot be expected to add or subtract, just so they focus on the process, may be a benefit to them.

I think this is most easily seen when one speaks in terms of **PARTS** and **WHOLES**. What is missing? In most of the earliest word problems it's one or the other.

If they think in terms of
PART + PART = WHOLE and **WHOLE - PART = PART** ,
the number sentences then become a choice between six variations on the above:

The most simple, **PART + PART = __** or **WHOLE - PART = __** ,

or the more difficult, **PART + __ = WHOLE** or **WHOLE - __ = PART** ,

and **__ + PART = WHOLE** or **__ - PART = PART** .

The first problem given as an example is more difficult because it is an additive situation that, given large enough numbers, requires subtraction to find the number answer.
517 + __ = 978 is not a question most young children are going to tackle.
978 - 517 = ___ is a question they will someday learn how to place in a vertical form so that a number answer can be found.

By practising word problems that result in each of the six types of incomplete number sentences, children can develop confidence.

They can learn that <u>additive situations sometimes require subtraction to solve</u>, and <u>subtractive situations sometimes require addition to solve</u>.

They can see that it is permissible to subtract sometimes when things are being put together, and add sometimes when things are really being taken apart.

It all depends on what is missing.

N.B. This does not mean that the first problems should be of these types. The first should be about additive situations that require adding to find the number answer, and subtractive situations that require subtracting.

This effort put into learning a process for handling most beginning problems has a carry over to **multiplication and division** problems later if it is done well. The previous comments apply to these as well.

In my Simple Math booklet I speak of

Number of Parts x Size of Parts = WHOLE , and

WHOLE -:- Size of Parts = Number of Parts .

Children can learn to recognize these situations in word problems when they see that **things are going together, or apart, in equal groups.**

Knowing that the **Number of Parts** and the **Size of Parts** are both **factors of the Whole, and that factors can change places with their partner in a number sentence** allows an even simpler representation:

FACTOR x FACTOR = WHOLE , or **WHOLE -:- FACTOR = FACTOR** .

Knowing this would allow one to see the possible word problems falling into types which are very similar to those for addition and subtraction:

The simple **FACTOR x FACTOR = __** , and **WHOLE -:- FACTOR = __** ,

or the more difficult **FACTOR x __ = WHOLE** , and **WHOLE -:- __ = FACTOR** ,

or **__ x FACTOR = WHOLE** , and **__ -:- FACTOR = FACTOR** .

Here children can be led to see that given two FACTORS, one multiplies to find the WHOLE, and given a WHOLE and one FACTOR one divides to find the other FACTOR.

It can all be handled much the same as with addition and subtraction word problems, **with one important difference.**

One FACTOR counts groups or parts in the world of the word problem.

The other counts the same objects as the WHOLE..

This can make it much more difficult to relate the number answer to the word problem and tell just what it is a number of.

Word problems are not always included in the math or arithmetic curriculum for young children. If they are they may not be given much time. But ,when they are there, I hope you will view them as something new to be taught, and you will consider the reading ability required to do them independently.

I haven't yet seen any math texts that really have problems written in language that most children would find easy to read.

Part 2,
Reading,writing, and understanding the value of larger numbers:

These topics remain important to us throughout our lives. It is amazing how easy it is for adults to be deceived when comparing or judging very large numbers. For example, when government officials and politicians speak in terms of millions and billions being spent out of taxes, or carried as debt, do we really know how much money this is?

A country like Canada with a relatively small population, roughly 30 million people, may have a debt of roughly 600 billion dollars. What does this really mean?

Look at it this way: 30,000,000 people owe $600,000,000,000.

If we take the same number of zeros off each we keep the same ratio.

This becomes: 3 people owe $60,000, or each person, no matter what age, owes $20,000, and that is on top of any personal debt they may have.

I’m afraid not all people of voting age are capable of dealing with what millions, and billions, and trillions really mean.

I hope you will read the following carefully to find those places that seem natural for adults, but can really be confusing for children. Perhaps you may find you have not fully understood the value(numerical quantity) of a number yourself.

In my class we had enough blocks (wooden 1in. (2.5 cm) cubes) to show an amount just above 1000. This was a great advantage. It allowed us to see the basic value of each of the first four places in our number system, and to see an emerging pattern.

Our **place value number system**, the system we use to represent amounts, is **based on groups of 10**. If we see this many x’s, xxxxxxxxxxxx, we write 12.

What we are doing is grouping them in 10’s, (xxxxxxxxxx) xx, and writing in a place value system that there is one set of 10 x’s and two single x’s.

For young children, there is something awkward about this system that adults may not think about because they do it unconsciously.

In many languages, but not all, we read from left to right, and young children have to be trained to move their eyes accordingly.

As we read large numbers we have to read from left to right.

As we write numbers we have to write from left to right.

But, when we judge what a particular number is to be called because of the place it is in, we have to make a judgment based on the number of places it lies to the left of the ones place, which lies on the right.

In other words the system requires us to be totally aware of a right to left sequence before reading from left to right.
The four place number my class formed with all their blocks was 1111, or 1,111.

This can be read as "eleven hundred and eleven" because many people read and write four figure numbers this way. This is something people may do, but it strays from the actual system.

Even the name 'eleven' does not follow the pattern through which most numbers are named. All the numbers from 11 to 19 are the same. They have two places, with the place on the right something other than 0. All the other two place numbers without a 0 on the right have a combination name. 41 is "forty-one". 69 is "sixty-nine". Those with a 0 on the right have a single name. 50 is "fifty".

People who write 1,111 tend to stick to the system. They would read this as "one thousand, one hundred and eleven".

This is further confused since some people also skip out the "and" in either way of reading the number. This isn't a big problem to us, but it could be for children.
My advice on this point is to stick to a system while the children are young and let them learn to stray from it later.

The Systems:
I hope you can see that writing, reading, and understanding numbers is not so simple.

<u>Writing:</u> This is the most consistent system. We first need enough unique numerals to name each single amount that can only be named separately. By numerals I mean shapes we form, and their names we use, to represent a particular amount. (2 and two)

Our place value number system is based on groups that could be matched one-to-one with this: (xxxxxxxxxx). We call that amount, "ten". All around the world other languages name it differently, but the amount remains the same.

We say we have a **base ten place value number system**.

We need then, a 0,1,2,3,4,5,6,7,8, and 9 as numerals to write with.

They represent the amounts that could stand alone when our system is applied. Beyond an amount of 9, we use those numerals in a special way.

You may be aware of other place value number systems. One system requires two numerals, 0 and 1. It is a base two system, using groups equivalent to (xx). Using this system you can write any amount with 0 and 1 that we can write using 0 to 9, but it takes a lot more space. However, it is ideally suited to computers that can represent the 0 by an electrical current of low voltage, and 1 by a slightly higher voltage.

Another system that has had limited use was based on groups of 12, and another in use is based on groups of 16. Both systems require additional single figure numerals.

In our base ten system, 10 represents the amount that is 1 more than 9. Children have to learn this is not a new shape.

It is a 1 and a 0 used in a very special way.

The 1 and 0 in 10 each represent what they did on their own. They **also** represent something new based on their **placement in relation to each other**. That is, 01 is far different from 10.

In 10, the 1, being one place to the left of the 0, now represents 1 group of ten, or a group equivalent to (xxxxxxxxxx).

0 has two very important functions.

The 0 in 10, being on the right of the 1, represents an empty group, AND serves to push the 1 into what we call the ten's place. 0 can act as a place holder.

In 01, the 0 really has no use, but we sometimes put it there. It does no harm.

My Simple Math booklet describes a number line made from adding machine paper that ran across the front and along most of one side. We moved a marker along it to indicate the number of days at school. This was numbered from 0 to 203 and allowed the children to always have a sense of where a number was in relation to others. It was not a crucial part of understanding the writing of numbers and place value, but it was useful.

In my Simple Math booklet I also described another activity we did every morning in my class. We had a board with three hooks, side by side, with numeral cards from 0 to 9 hanging on each hook, 0 being at the front to start. The set on the right was the one we began using on the first day of school to record the number of school days.

I kept the 0 in the middle set and left set facing the children as we went through the cards on the right, putting yesterday's numeral to the back of its pack to expose the new one. On the first day we had 001.

On the tenth day the middle zero was sent to the back of the pack and 1 was exposed. Of course, at the same time the 0 on the right appeared again.
We had 010 and we had been in class for ten days.

On the 40th day of school my class would have seen 040 on our display.
The 0 on the left stayed where it was until the 100th day.

Children need to be reminded often that 0 serves two important functions in our place value number system. **0 represents an empty set, but it also serves to fill a place.** There is a big difference between 1,000,000 and 10 , and that difference is made with 0's that force the 1 a special number of places to the left.

At this point I should remind you that young children may not have a well developed sense of right and left. Even up to age 8 and beyond, children will mix up 'no' and 'on'. It can happen to adults. Just recently I received a lengthy fax message that wasn't intended for me, 22 pages of confidential information. I looked at the phone number of the party it was meant for and tried to phone them. But, I made the same mistake as the fax sender. My phone/fax number starts 531, and the fax should have gone to 513. I saw this, but when I typed it into my computer phone, I typed 531. Luckily, there was only an answering machine to receive my message about an errant fax, and a correction when I realised my mistake.
This was the same mistake the original sender had made, a reversal of numerals.

My wife and I arrived home from a trip recently to find an elderly, respected criminal lawyer, and politician, had left a message on our answering machine containing confidential, and not flattering, information for another lawyer about one of his criminal clients. He had mixed up some numbers when he dialled, but was confident enough in his own abilities to leave a message. It could have been a very serious mistake.

Now if adults can make this kind or error, certainly we can expect children to.

Our left to right reading and writing is not something we are born to do.
Other people arrange their symbols in other ways and get along just fine.

Children have to develop this left to right movement we require to read and write. To some degree this requires them to be physically aware of their own left and right sides, and that of others, and of other objects.

We spent time in my classes practising just this left and right orientation and I will elaborate on these activities in another booklet.

In my Simple Math booklet ,p. 5, and my Games booklet, p.9, I've described activities where children have to decide the larger or smaller of 40 and 04. This is not easy for some. It requires a confident sense of left and right that not all have, but practise pays off for them.

Now, going back to my class counting the days at school, when we got to the ninth day at school, we had to start thinking about place value.

On a ledge above this display I had put one block for each day. These were plastic blocks that snapped together, but they were left loose at this time.

We could see that there were no more numerals in the card set we had been changing. We had 009. Hanging behind the 9 was a 0.

The next day I introduced the idea that we could put a set of things together and call them 1 group. We would have a special rule for naming the number of days from now on. Every time we got a set like (xxxxxxxxxx) we would call it 1 group of ten.

This meant our set of plastic blocks now were snapped together in a line.

That 0 in the next place to the left would go behind to expose a new 1, a 1 that was special and didn't mean just what it had on the first day of school. The 9 on the right went behind to let us see the 0 again.

The blocks showed us what had happened. All the blocks had left the one's place and were now being counted in a new way in the ten's place.
Thereafter we referred to the numerals we put up as going into the one's or ten's place. This time the one's place had nothing in it, as shown by the 0.

We had 010. The 0 on the right, filling the one's place, gave the 1 its special value.

On the chalkboard I left this reminder: **1->|<-10** . (Ten could be shown as 1 in the ten's place, but that group of 1 ten could be broken up to make ten separate ones.)

The following day was great for demonstrating the effect of a place giving a numeral value. We had 011. Each 1 was a **"1"**, but that on the left was special. It was **1 group of ten**. We had 1 group of ten, and a simple 1. The blocks above demonstrated this. We had two 1's of different value, yet they were both 1.

The day after was the introduction to the idea that a 1, **because of its place**, could be worth more than a 2. The blocks helped us see this. We had 012.

This idea was reinforced through to 019, but some children were still uneasy about that 9 being worth less than the 1. It's difficult for some children to break away from their early learning about what is big and what is small, and come to see that big and small can depend upon the placement of a numeral.
The blocks above 019 helped.
(This difficulty persisted for some time, with some having trouble understanding that 21 was bigger than 19, or 68 was smaller than 71. They could see it if given real objects, but when looking at the two written numbers they weren't sure.)

When we got to the twentieth day we reviewed the notion of (xxxxxxxxxx) being called 1 group. We now had enough blocks to demonstrate this again. We had 020.

This went on until we had 099. Everyone knew what number came next, but not everyone knew how it all came to be.

On the hundredth day we found we had 10 sets of 10 blocks. We reviewed the rule that any set of 10 was grouped together to become a new set of 1. Luckily these blocks could be placed so that all the sets of ten joined. They formed a square.

(I added some tape to the back of them just to make sure they stayed together.)

Obviously there were no separate blocks, so the 9 in the one's place was put to the back of its set so 0 would show. There were no separate sets of ten blocks, so the same thing happened there. We went to the third place to the left, put the 0 to the back, and we had 100.

This place that had waited so long to be used was the hundred's place.

Most years all the primary grades celebrated this day, but I also celebrated the 111th day in my class. I called it **Place Value Day**. On that day we had 1 in each place, and with the blocks it was easy to see that each 1 counted something different, and that the value of the 1 on the left was far more than the 1 on the right.

One of the activities for this day was to use other wooden 1 in. (2.5 cm) blocks we had to form sets of 1000, 100, 10, and 1. (Luckily, we had a lot of blocks.)

We also did the same with pegs in pegboards, with room for 100 in each. We stacked 10 full pegboards to show 1000. We set one full one of 100 to the right, one with a single row of 10 to the right of that, and one with 1 peg in the last place on the right.

At this point I'll add an activity also described on p. 30 of my Simple Math booklet.

My class made a simple card to mark the 111th day. A sheet of paper folded twice formed the card.

The front of the card had 111 **written in large numerals,** and in a **diagonal line** down the page, starting at the top left.

The children wrote **t 1 o , m 1 y, fam 1 ily** (or mom, or dad,...) on the front.

On the inside left the children made 10 rows of 10 **x**'s.

On the inside right I had already duplicated a diagonal line of 10 large **O**'s.

Inside the **O**'s they printed "I love you", which, if you count the spaces, uses all 10 **O**'s.

The children colored and decorated the cards as time allowed and filled in their names on this preprinted section to the lower left of the circles:

"From ________,
111 days at school today!"

They liked the idea of taking home 100 kisses (x's), 10 hugs (O's) and 1 "I love you". (One French Immersion teacher made this work in French.)

But, going back to the 1,111 wooden blocks, we kept these on the floor for many days where we could gather together to discuss them.

Remember that the blocks used were 1 in. (2.5 cm.) cubes. I believe these were a very good size for what we were to do next. We did have 1 cm. cubes, but these were too small to be handled easily, and too small to provide a lasting image in children's minds of the rate at which the value of 1 increases as it moves to places to the left.

I think it's important for everyone to have some image that allows them to judge the relative value of numbers.

These blocks provided a way to build a simple picture in the mind of what happens as we go to the left from the one's place, a picture that remains consistent no matter how large the number is, and one that is consistent with our method of writing large numbers. (It also operates to the right of the one's place.)

We had **1,111** blocks. Some would write **1111** , but that's not my preference.

The **1** on the far right was represented by a single block, a single **cube**.
(That cube was important to the picture we were developing.)

The **1** in the 10's place to the left of it was a **line of 10 cubes**.

The **1** in the 100's place to the left again was a **square of 100 cubes**.

The **1** in the 1,000's place to the left again was a **CUBE of 1000 cubes**.
With a little imagination it was easy to see that a pattern had started.

To show 1 in the 10,000's place we would need a **line of 10 cubes** of 1,000.

To show 1 in the 100,000's place we would need a **square of 100 cubes** of 1,000.

To show 1 in the 1,000,000's place would take **cube of 1000 cubes of 1,000.**

A **pattern** had formed that hinted at the value of the commas in the numbers:
square line cube, square line cube, square line cube, square line cube ...

We discussed the amount of space each of these large numbers of blocks would take up. (I've added approximate measurements. With the children, we just used our eyes, comparisons with other objects, the room, the school, and the playground.)

The thousand cube was 10 in. (25 cm) on a side, so the 10,000 line, having 10 such cubes, would be 100 in. long (8 ft. 4 in. or 2.5 meters), 10 in. wide and 10 in. high.

The 100,000 square would cover the floor for 100 in. on each side and be 10 in. high.

The 1,000,000 cube would be the 100,000 square stacked 10 high and would be 8 ft. 4 in. (2.5 meters) on each side. In other words it would be close to the ceiling.

Older children might like to work out the weight of such a cube. We wondered if the floor would support its weight. **Obviously a million is a lot of blocks, and we did learn the word 'million'.**

We went on to think about 10,000,000. That would be a line of 10 of the cubes holding a million blocks, a line 83 ft. 4 in. (25 meters) long, 8 ft. 4 in. wide, and the same high. This would have to be made outside.

100,000,000 blocks would form a square of cubes of a million blocks each, 83 ft. 4 in. on each side, and 8 ft. 4 in. deep.

1,000,000,000, or a billion, blocks would form a cube, 83 ft. 4 in. (25 meters) on each side. **Obviously a billion is far, far bigger than a million; a 1000 times bigger.**

10 billion would form a line of billion cubes 833 ft. 4 in. long, 83 ft. 4 in. high and wide.

100 billion would be 10 such lines side by side and form a square of a billion cubes, 833 ft. 4 in. on each side, and 83 ft. 4 in. deep.

(My country's debt would be a stack of about 6 such squares, if each 1 in. cube represented one of our dollars.)

A trillion, or 1,000,000,000,000 , would be a cube formed by a stack of 10 such squares, and would be 833 ft. 4 in. on each side.

There is a system at work here. Every time a 0 is added to the right of the 1, or any number, it becomes worth ten times more.

A ten is ten ones.
A hundred is a hundred ones, or ten tens.
A thousand is a thousand ones, or ten hundreds.
A million is a thousand thousands.
A billion is a thousand millions.
A trillion is a thousand billions.

When children begin to multiply they can see this as:

10 = 10 x 1
100 = 10 x 10 x 1
1,000 = 10 x 10 x 10 x 1
10,000 = 10 x 10 x 10 x 10 x 1
100,000 = 10 x 10 x 10 x 10 x 10 x 1
1,000,000 = 10 x 10 x 10 x 10 x 10 x 10 x 1
1,000,000,000 = 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 1
1,000,000,000,000 = 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 1

Probably I’ve gone on with this longer than you would wish to, but I’ll end this with a note that once you get to decimals, to the right of the one's place, you will find there is a mirrored symmetry centred on 1.

To the left of 1, each numeral represents a number of 10’s. To the right of 1, each numeral represents a number of 1/10 ths. Instead of each place becoming worth 10 times more for each move to the left, t<u>he places to the right are worth 10 times less.</u>

Before leaving this section on writing numbers, and understanding their value, you should note that I have used commas to divide the large numbers into groups of 3 places. This is not always done, but it is an aid to reading, and understanding.

Remember, children have learned to scan to the right. Understanding and reading a number requires one to judge a number of places from the right back to the left.

Commas help.

Writing large numbers spoken by others, or thought of, is aided by the knowledge that a comma goes after each new name beyond ‘hundred’. There is a pattern.
No matter how large the number, it has a hundred's, ten's, and one's place, followed by a name such as trillion, billion, million, thousand. Only with the smallest three places do we not add a name. It is understood to be ‘ones’.

<u>Reading Numbers</u>:

There is a system to reading numbers, but it isn't so simple and consistent as that for writing numbers.

First, the numerals 0 to 11 have unique names: zero, one, two, three, four, five, six, seven, eight, nine, ten, eleven.

Twelve has a hint in of two in it as it starts the same, as does twice. Thirteen also starts like three, and like third. Fourteen has four in it, as does fourth. Fifteen hints at five through fifth. Sixteen, seventeen, eighteen and nineteen all have their number of ones in their name. From thirteen on the ‘teen’ hints at the ten in the number.

Learning to read these numbers is no easy task for young children, even if reading them only implies knowing what to say when looking at the numerals, as 17.

Then we get to twenty and the task becomes much simpler as a real system starts to operate.

We now have to learn twenty, thirty, forty, fifty, sixty, seventy, eighty, and ninety.
All of these are related to what we’ve learned before.

We also have to learn that after each of these names we can use all the names from one to nine before we go to the next new name.

(A counting chart with a first row from **0** to 9, a second row of 10 to 19, and so on, is a great place to note how all the first row gets used over and over again.

Children like to look for patterns, and its easy to see the columns of 0, of 1, of 2, etc., beside the changing numerals in the ten's place. It's also easy to look down these columns and see the numerals from 1 to 9 appearing beside each 0, 1, 2, ...

I liked to start my first row with 01,02,03, ... This way the numerals 0 to 9 in the ten's place appeared before each 0 to 9 in the one's place.)

Then we get to the hundred's place. Children are usually delighted to learn how far they could possibly count once they have mastered everything to one hundred.

There are no new names until we get to one thousand.

There's only a flexible procedure. We may say "one hundred and one" , which I prefer, or "one hundred one" for the next number. We just repeat everything from one on to ninety-nine. After that we say "two hundred" and begin all over again.

Eventually we get to one thousand.

Here it can get confusing.

We can continue on in the same way saying, "One thousand, ...", and add all the names we have used up to one thousand. Then we can say "two thousand, ..." and so on using all the numbers before 'thousand' that we used up to one thousand, and all the numbers after 'and' that we used up to one thousand.

But when the number has four figures, some people choose to break from this system. They would read the two largest places as a two figure number and put 'hundred' after it. Then they would add the last two figures.

4176 would be read as, "forty-one hundred and seventy-six".

To be consistent with the way in which most large numbers are read this would be, "four thousand, one hundred and seventy-six", and would be written as 4,176.

This only happens when there are four figures, but it can be confusing for young children.

After conquering all of the thousands, we get to the million mark. That is handled just the same as the thousands numbers, as would be the billions, and so on.

<u>Comparing numbers</u>:

As I mentioned before, some children have difficulty judging the relative size of numbers. Which is bigger, 35 or 29? Which is smaller, 28 or 42?

Some will have difficulty deciding between 42 and 24, or 56 and 65.

These children need to really focus on the value each numeral has in its place.

They may have difficulty seeing the numerals as separate entities, each with its own value. They may have trouble deciding which numeral is on the left, or the right.

Take 35 and 29. Children have to learn that the place that decides their relative value is the place that is worth the most, the ten's place. The one's place has no bearing on the question, "Which is bigger (smaller)?" They have to come to see that it's a question of which number will be worth more (less), one with 3 tens or one with 2 tens.

Counting real objects can make this point. I liked to use long dozen type egg cartons <u>with two spaces cut off</u>. Any small objects could be counted this way and grouped clearly into sets of 10 with the ones remaining ungrouped.

The number line we had stretched along two walls of our classroom also helped. The larger number was farther on, the smaller was farther back.

Our number chart helped in the same way. The larger number was farther on or down, the smaller was farther back or up.

As I also mentioned before, we had activities where we practised judging the larger or smaller number, or where we intentionally formed a larger or smaller number with two given numerals.

I've spent a lot of time talking about something that doesn't get a lot of space in math texts. I did so because, in my experience, children can have real problems working with larger numbers simply because they have not completely come to terms with the meaning of place value. They don't have the ability to look at a two figure number and see it as one amount in one sense, and as two separate amounts in another.

<u>That is they can't comfortably switch between 43 and 4 tens, 3 ones.</u>

These children have difficulty with the more complex two figure adding and subtracting questions, and they certainly will have difficulty with multiple figure multiplication and division, particularly division by two or more figures.

Time spent on understanding numbers is time well spent.

<u>Part 3, More than, Less than, and Differences:</u>

This seems to be a subject that goes in and out of favor. There were times during my teaching years when we did teach > (more than) and < (less than) at some point during the primary grades. There were times when it was not required.

The concept of more than and less than is an important part of understanding number. It can provide practice of place value knowledge, practice with addition and subtraction number sentences, and an introduction to a special subtraction situation.

I'm sure most teachers of the youngest children have come across a little story, or made up one, to teach the difference between the symbols > and < .
I like the greedy crocodile's mouth myself. This crocodile always opens his mouth towards the bigger number. Then there's the big bird that does the same thing.

On p. 8 of my Games booklet you'll find an activity that requires young children to practise the ideas of 'more than' and 'less than'. In the section before this I've already mentioned other activities and games requiring the same place value judgments.

Before starting to teach this work, I think you should make children aware that what is being done falls under the general category of comparing two of anything.
That is, you can say: Bill is taller than Tom. My coat is lighter than yours. This apple is sweeter than that one.

Of course you can say these things the other way around: Tom is shorter than Bill. Your coat is heavier than mine. That apple is not as sweet as this one.

Note that , with the exception of one case, all the comparison words end with 'er '.

It is important that children see you can go both ways.
It is also important they see that the comparison is <u>only about one particular quality.</u>

The quality we wish to judge here is number size, or value.

This work really can start with the youngest children. 5 > 3 is not a hard concept.
It may take awhile before they are comfortable changing this to 3 < 5.

Later, children can be asked to write a comparison statement about larger numbers, say 59 and 61. This is a set where some children have difficulty. They see the 9 and think the number with 9 must be bigger.

It also requires them to make the shift from the fifties to the sixties.

I didn't make a point of this in the previous section, but children do have the most problem judging relative size where one is close to the next amount of tens, and one is just over it. It is also the place where there can be the most difficulty in learning to count.

Asking them to use > and < with 59 and 61 brings out this problem so it can be dealt with.

With objects to count at first, a number line and/or 100 chart to use later, this is not a hard skill for most children to master.

<u>Differences</u>:

At some point one can say to children, " Yes, 5 is more than 3, but how much more?" "3 is less than 5, but by how much?"

One has to think of these questions in terms of real things. There are two simple but different situations where you might ask these questions.

Sally has 5 cookies in her lunch. Jane has 3. Who has more cookies? How many more?
ooooo
ooo
Put these separate sets side by side and you can see the difference.

Jim and Sam walk home together. Jim walks 5 blocks, and Sam walks 7. Who walks farther? How many blocks farther?

_ _ _ _ _ _ _

This time the count is along the same line, as it would be on a number line.
(_ _ _ _ _) _ _

There are a number of strategies for working these out when they aren't obvious.

One can count up from the smaller amount to the larger one, or the reverse.

In the cookie example, one can take a group of the larger one equal to the smaller one and then count to see how many there are beyond that amount.

One can also go to the smaller set and count on enough imaginary cookies to make the larger set.

In the number line example, one can count on from the smaller distance to the larger, or the reverse. **Make sure children are counting steps, not numerals.**

At some point children can learn to subtract the smaller from the larger to find the difference. This is a more abstract activity.

If children are familiar with number sentences they can reason out that the smaller amount plus the difference equals the larger amount, as 3 + __ = 5.
Or the larger amount minus the difference equals the smaller amount, as 7 - __ = 5.

That is if you take away the difference from the larger, or add on the difference to the smaller, both would be equal.

5 > 3 , by 2 , or 7 > 5 , by 2, and 3 < 5 , by 2, or 5 < 7 , by 2 were the type of statements I liked children to be able to make as a final answer.

With larger numbers, 61 > 49 , by 12 , since 61 - 49 = 12 might have been more appropriate.

At some point you will like to bring in comparisons with three or more numbers. Here the comparison words change. With 3, 7, 9, the larg**est** is 9. 3 is the small**est**.

This can provide good review: 3 < 7 , 3 < 9 , 7 < 9 , 9 > 7 , 9 > 3 , 7 > 3.

This might be shortened to: 3 < 7 < 9 , or 9 > 7 > 3.

Another set of questions can be more open: 3 > _____ , or 5 < __ , __ , __ , or
5 > ______ < 3.

Part 4, Measurement:

In this section I'd like to look at those places where we have to group things according to a rule. Mainly this is when we measure.
I'd like you to see that this is an area that needs careful attention.

We've already dealt with this under place value. Here we have a fixed rule:

1->|<-10.

If we get a set of 10, we group it and count it as 1 in the place to the left.
Sometimes we have a need to go the other way, as in questions where the 1 on the left may be changed to 10 on the right to allow subtraction to take place.

This **1->|<-10** rule is a very common one in many countries.
It is the basis of the metric system, and decimal based monetary systems.

This simple rule is not always held to as we count certain types of things, and this can create problems for children. Dealing with other rules can reinforce the concept of grouping in order to count in larger sets under a different name.

Usually children's texts will not ask them to deal with conversions, but some may be part of their lives.

Take eggs, for example. We normally group them in sets of 12 and name those sets as a dozen.

12 is a number that has advantages over 10 for packaging. It can be efficiently arranged in two ways, although most often we see eggs in cartons of 2 rows of 6.

It can also be evenly divided in more ways than 10.

Commerce has seen the use of dozens in packaging many other things. For large amounts it is common to find a dozen dozens, as in a shipment of pencils.

Children now love to hear the name given to 12 sets of 12, a gross.

<u>Units of time can be a problem</u>.

To go from seconds to minutes we use a ratio of **60 to 1.**

To go from minutes to hours we use the same ratio, **60 to 1**.

To go from hours to days, we use **24 to 1**.

But most of our clocks only show hours up to 12, so children have to learn there is a morning set of 12 which starts at midnight, and an afternoon set of 12 which starts at noon.

To go from days to months children have to learn this is flexible, but **30 to 1** is often used.

To go from days to years the ratio is usually **365 to 1**, but every fourth year this becomes 366 to 1.

To go from months to years the ratio is fixed, **12 to 1**.

After this last, it seems we finally go back to a ratio of **10 to 1.** **10** years become a decade, 10 decades become a century, and 10 centuries become a millennium.

(A long time ago at least one nation toyed with changing to completely metric time.)

When it comes to measuring distance or length, weight, and volume, we in Canada have a unique problem.

Many, many years ago our government decided it knew that what would be best for us was to switch to the metric system, despite the fact that our largest trading partner was still using, and continues to use, measures based on the English Imperial systems.

Now we have to learn to operate in both systems, the old and the new.

For length we deal with inches to feet, a ratio of **12 to 1**, inches to yards at **36 to 1**, and feet to yards at **3 to 1**. These are the measurements that are people sized.

Then we have feet to miles at **5280 to 1**, or yards to miles at **1760 to 1**.

In the metric system we have millimetres to centimetres at 10 to 1, centimetres to decimetres at 10 to 1, and decimetres to metres at 10 to 1. (Decimetres are seldom used.)

Then we go from metres to kilometres at 1000 to 1.

Still we see children's rulers measuring 30 cm. long since this is about equal to 12 in., a handy length to use. In fact many rulers show both inches and centimetres and children have to learn to use the proper scale.

Classrooms will be equipped with metre sticks, and sometimes yard sticks, for longer distances. 3 rulers may make the same distance as 1 yardstick, but they won't equal one metre stick.

Ask children their height, or adults, and they will probably answer in feet and inches.

Of course these units of measurement are also linked to measurements of area, and volume. Such forms of measurement each have their own unique names and ratios to deal with as well, but young children seldom have to deal with them.

The measurement of weight also uses two systems. Officially ours is the metric one, with grams and kilograms. The reality is that when grocery shopping one finds both in use. Many people feel the gram is too small and the kilogram is too large. Meat especially is often labelled as so much per pound, and so much per kilogram. Besides that, a lot of food items come from places where ounces and pounds are used.

Ask Canadian children or adults what they weigh and their answer will likely be in pounds. I know mine would be. Bathroom scales don't wear out that quickly.

In any event we have to be prepared to switch between ounces and pounds with a ratio of **16 to 1**.

Grams and kilograms, as the name indicates, have a ratio of **1000 to 1**.

Young children won't have much to do with the larger units of both system, which is just as well as they can be confusing.

Liquid volume is another common type of measurement where attention needs to be paid, and it is even more confusing. (The flight of a large jet full of people almost ended tragically because of an error in converting the measurement of fuel on board.)

In Canada, we used to, and to some extent still do, use the Imperial system.

We measured liquid volume with ounces, cups, pints, quarts, and gallons.

Ounces to cups had a ratio of **8 to 1**. Cups still are commonly used in cooking.

Ounces to pints had a ratio of **20 to 1**.

Pints to quarts had a ratio of **2 to 1**.

Cups to quarts had a ratio of **5 to 1**.

Ounces to quarts had a ratio of **40 to 1**.

Quarts to gallons had a ratio of **4 to 1**.

<u>In the U.S.A</u>. the same words are used, but they don't always mean the same.

Ounces to cups stay the same.

Cups to pints became **2 to 1**, which meant a U.S. pint is 16 ounces, instead of 20. Pints to quarts stayed **2 to 1**, so a U.S. quart became 32 ounces, instead of 40.

Quarts to gallons stayed the same, so a U.S. gallon had 128 ounces, not 160.

Shopping for liquids in Canada means one can find containers measured with the metric millilitres and litres, with a ratio of **1000 to 1**.

One can also find containers holding an Imperial pint or quart, or a U.S. pint or quart. Large quantities, such as pails of paint, may come in Imperial gallons, or U.S. gallons, or litres.

Ever since this government's decision to switch measuring systems there has been a lot of money expended for charts, pamphlets,gadgets, and even calculators to help citizens make the switch between the metric system and the Imperial system.

Despite all these we still have puzzles to work out. In the U.S. for example, cars are rated as achieving so many miles per gallon of fuel. In Canada, our gallons are different, our cars measure distance in kilometres, and our gas stations sell fuel in litres. Canadian cars are rated as using so many litres per 100 kilometres.
If you want some work, try comparing the reported mileage of cars from each of the two countries.

Following recipes can also be a challenge, as when the recipe calls for so many ounces of chocolate chips, and the stores only have bags measured in grams.

Besides just becoming familiar with the units we use to measure, you may ask children to convert some of the simpler measures from large units to small units, or the reverse.

If you were working with cups and quarts for example, you could ask how many quart jars would be filled by 12 cups of water. Depending on which quart, you would be asking how many times you could get 5 cups out of 12, or 4 cups out of 12, and how many would be left over.

You could ask how many cups you could get from 2 quarts.

For the youngest children these examples would require working with real objects.

I have not seen a text that spends a lot of time with converting units of measure. Still, working with measurement provides practise opportunities for children who have begun multiplication and division, and for those working on multiple figure addition and subtraction, which will be the subject of Part 6.

3 metres, 5 centimetres	3 m. , 18 cm.
+2 metres, 8 centimetres	- 1 m. , 9 cm.

These questions require one to work in columns, keeping the like units together, just as required by the more abstract work in Part 6.

One last topic in this area is money. Money in a lot of countries is related to decimal fractions. I don't intend to say more than this: there is an opportunity to practice changing from one money form to another, and to relate the patterns of our place value system to the right of the one's place where we get a mirror image of the left side.

Part 5. Fractions:

Math or Arithmetic texts I have used seldom gave much space to fractions, and usually it was only fractions of solid objects such as cakes, pies, and pieces of paper. The emphasis seemed to be on providing a visual image of particular fractions.

Fractions are not such a simple subject. They cause a lot of trouble to students in later years so whatever can be done with them in the early years will be appreciated.

A fraction is a special kind of number that we use to talk about a quantity that is less than one, that is, where something is a portion of a whole.

There are two basic fraction situations, one where a fraction is used to name a part of a whole one, two, or three dimensional object,
and the other where a fraction is used to name a part of one group of objects.

Common fractions have two numerals each, one on top of the other, as **7 / 8.**

I taught children primary grades that **the number on the bottom, 8, counted the number of equal parts the whole was divided into.**
The number on the top, 7, counted the same parts as the bottom number, but only those being talked about.
The bottom number described the size of the parts, the top number counted them.

I did mention that the top number was called the **numerator**, and the bottom number was the **denominator**. I tried to relate these names to their jobs.

Most of the time we did not use these names. We talked instead of **the counting number, and the describing number**. These were more familiar words.

By telling us how many parts there was in one whole the bottom number let us know what the parts would be like. It let us picture them.

This was the number we had to focus on first.

We talked about a party where there was just one cake. How big a piece of that cake would you get if you invited one person to share it, or 4, or 10, or 100, or 1000?

What if you invited no one?

We did the same with a plate of 8 cookies. How many would you get if you shared them with 1 other person, or 3 others, or 7 others? What if you didn't share?

Changing the bottom number, the describing number, changed the amount 1 portion would be. As the describing number got larger, each portion got smaller. The smallest number, 1, would mean the portion was the whole thing.

The top number was there to tell us how many of the bottom number's parts we were using the fraction to talk about. It counted just as we were used to counting, only it counted special equal parts of a whole.

It counted the kind of parts the bottom number told us about.

When this top number, or counting number, got bigger it meant there were more parts. (but the parts were always the size described by the bottom number)

What if a mother put a cake on the table for her child to eat some of after school, and she had cut it into 8 pieces. If her child ate 1 / 8 of the cake there would be a lot left.

But what if her child ate 2 / 8, or 5 / 8, or 7 / 8, or 8 / 8?

By discussing such things, and by cutting up real objects and pictured objects, the children got a sense of what a fraction was about.

A fraction is a single number in one sense, but to understand it we have to consider both numbers used to form it.

There may be emphasis in texts placed on knowing what a quarter, a third, and a half are like. There may be work done on relating a quarter to a half, or to a whole.

What I think was lacking in the texts and curriculum I had to use was an effort to promote a real understanding of why 1 / 4 was the size it was, and why 1 / 3 was larger than 1 /4.

For this reason I concentrated on the meaning of the two numbers, on what they were telling us, rather than just picturing how big an amount the fraction was as a single number.

I wanted to look at the **three** and the **four** in 3 / 4, not just the **three-quarters**.

In a sense this was like looking at the value of the four and the eight in 48, rather than simply the forty-eight.

Using squares and circles on the chalkboard, and sets of real objects, my class worked on understanding just what fractions meant.

I might put up a square, and the fraction 3 / 4. I would ask how many equal parts I had to divide the square into. Then I would ask how many of those parts had to be made different in some way.

I might put out 8 blocks, and the fraction 3 / 4. I would ask someone to divide the blocks into the correct number of equal groups. Then I would ask how many of those groups was the fraction counting.

Working like this we were able to compare fractions. The square divided into 4 could be divided into 8. We could note that each of the former parts became two smaller parts. If the 3 / 4 was marked out, we could see it also had 6 / 8 so we knew that these two fractions talked about the same amount, but in a different way.

This idea could be extended so we could see there were many fractions that could mean the same amount of a whole, yet look very different from one another.

These were equal fractions. 6 / 8 = 3 / 4 .

It wasn't hard to see that there was the same kind of relationship between the counting numbers in these fractions as there was between the describing numbers.

This was especially true if we did 1 / 2, 2 / 4, and 4 / 8. When we made the describing number twice as big, the counting number became twice as big.

We could also see that 7 / 8 was larger than 3 / 4, and 5 / 8 was smaller, but it was easier to see this if we changed the 3 / 4 to 6 / 8. Then we had 5 / 8, 6 / 8, and 7 / 8.

For children who knew a little about multiplication and division it wasn't difficult to learn how to change fractions so they could be easily compared as to size, that is changed to a **common denominator.**

This need to convert fractions to a common denominator is one area where children have difficulty later on in school when they begin to add and subtract fractions.

I have taught this long ago while teaching grades 4 to 7, and I can remember it was very difficult work for some children.

There are things we can do in the primary grades to make this work easier. We can start to teach fractions in Kindergarten. I don't mean just the names, as one-half, and one-quarter. We can start to teach the meaning of the parts of fractions.

We can create a situation where there is sharing to be done. It can be real or the children can pretend it is real.

Two children have a big cookie to share. How many pieces should the cookie be cut into? What should we be careful to do as we cut the cookie? Is each child going to feel their share is fair?

Each child has a piece of the cookie. That piece is one piece out of two pieces in the whole cookie. Both pieces are the same size.

We say each child has half a cookie. That is written with numbers as 1 / 2. The bottom number tells how many pieces the cookie was cut into. The top number tells how many of those pieces each child got.

This focus on the meaning of the two numbers in a fraction can be started at an early age.

Over the next three years the world of fractions can be developed.
<u>I'm not trying to say that fractions become a big topic, like the one to follow this section, but they can be brought in here and there throughout every year.</u>

Asking questions such as,
"How many children are in our class?",
"How many boys are there?",
"How many girls are there?",
"What fraction of the children in our class are boys?", and
"What fraction are girls?",
can provide opportunities to explore the meaning of the two numbers in a fraction.

Say the answer to the first question is 30. This becomes the bottom number since it tells how many parts are in the whole.

If the second answer is 17, then the fraction that are boys is 17 / 30 since the 17 on the top tells us how many of the 30 in the class we are talking about with this fraction.

If the third answer is 13, then the fraction that are girls is 13 / 30 since the 13 on the top tells us how many of the 30 in the class we are talking about with this fraction.

We can notice that the two top numbers in these fractions add up to the number in the whole class.

We can ask questions about fractions relating to many things to do with children: hair, shoes, brothers, sisters, pets, ...

There are many objects in a classroom that provide the same opportunities: windows, desks, walls, light fixtures, ...

When children begin to multiply and divide there are other opportunities to start working with fractions.

I have had second grade children learn to do questions such as 1 / 4 = __ / 8 after they have started multiplication and division.

They can think of 4 x __ = 8, and then 1 x 2 = 2 to see that 1 / 4 = 2 / 8.

Fractions can exist on their own for us to work with but when they are really used they are related to something else.

I have had them find fractions of a group of objects. This required them to divide first, and then multiply.

I would have asked them to find 3 / 4 of 8. First they had to divide 8 by 4 to find that there were 2 in each part. Then they had to multiply that 2 by 3 because the counting part of the fraction said there was to be 3 of those groups of 2. They would find that 3 / 4 of 8 was 6.

Now to talk about how work done in the primary grades can be of help later on.

In my Simple Math booklet, starting on p.19, I discuss the use of factors in multiplication and division. Then, starting on p.28, I discuss the use of factors in working with fractions, particularly the addition and subtraction of fractions where one must find a common denominator, hopefully the LCD or lowest common denominator.

A common denominator is simply a way of saying that two or more fractions are talking about exactly the same sized parts or groups. The lowest common denominator would be one where the parts or groups are as large as possible, yet the fractions will have parts of the same size.

This may be where some children have difficulty.

When adding or subtracting fractions, what they really add or subtract are the counting numbers on the tops of the fractions, the numerators, but those numbers must be counting the same types of things.

It wouldn't make sense to say that we were adding 3 apples and 4 oranges to get 7 apples. We have to find a common name. It would make sense to say we are adding 3 pieces of fruit and 4 pieces of fruit to get 7 pieces of fruit.

Unfortunately finding a common name for fractions is not so simple.

As we change the part of the fraction which names through description, the denominator, we also change the counting number, the numerator.

As well, those numbers can only be changed through multiplication and division to a unique set of numbers if they are to be useful to us.
This is where a working knowledge of factors, and multiples, can help, and very young children can learn to use them.

Factor, as a name, is a relationship word. It tells us that a number will divide evenly into another, using whole numbers. 3 is a factor of 6, but not of 7.

Multiple is the name related to 'factor'. It tells us a number can be created by multiplying a whole number factor by itself or another whole number factor.
6 is a multiple of 3, but 7 is not.

Factors and multiples are directly tied to working with fractions.
They determine what the describing part of the fraction can be changed to.

If 3 / 8 and 4 /12 are to be added, we must find a number of parts that both bottom numbers can be changed to before we can add the same kinds of things.
A knowledge of factors and multiples would tell us that the smallest number both could be changed into is 24.

8's multiples are 8, 16, 24, ... , or 8 is a factor of 8, 16, 24, ...
12's multiples are 12, 24, 36, ... , or 12 is a factor of 12, 24, 36, ...

Each of the 8 parts in 3 / 8 must be divided into 3 parts so 8 x 3 = 24 does this for us.

Each of the 12 parts in 4 / 12 must be divided into 2 parts to give us 24 in a whole and 12 x 2 = 24 does this.

But, when we change the bottom number we also change the top number because it counts the parts in the bottom number. **What we do to the bottom we do to the top.**
So, if we've done 8 x 3 = 24, we must also do 3 x 3 = 9. 3 / 8 becomes 9 / 24.
If we've done 12 x 2 = 24, we must also do 4 x 2 = 8. 4 / 12 becomes 8 / 24.
These can be added by combining the counting numbers, 9 and 8.
The answer is 17 / 24.
(We don't add the bottom numbers because they are just like names.)

You have probably noticed I used 4 / 12, a fraction not in its simplest form. This is another problem area in later grades. **By simplest form we mean that the denominator has been made as small in number as it can be**, or the parts it names are as large as they can be, **and still have the counting part of the fraction, the numerator, be a whole number.**

Changing a fraction to its simplest form requires factors and multiples again.
12 can be made from 1, 2, 3, 4, 6, and 12.
4 can be made from 1, 2, and 4.
4 is the biggest common factor, and it lets us make the smallest common denominator.

If we make the 12 into groups of 4 we will have 3 groups. The 4 in 4 / 12 can become 1 group and still count the same amount it did before.
4 / 12 = 1 / 3.
If you add the 3 / 8 to 1 /3 you will still get the same answer, 17 / 24.
In my Simple Math booklet, starting on p.28, I also talk about **prime and composite numbers**, and the way **prime factors** can be used to help find the least common denominator (LCD) of larger denominators. I won't go into that here.

Prime and composite numbers were also a part of my work with multiplication and division, a part that can be valuable in later grades.

There are other fraction topics that can be started in the primary grades.

We talked about and did a little work related to proper fractions and improper fractions.

Proper fractions are those that really do talk about a part of something. The counting number on the top in these is always going to be less than the describing number on the bottom. All the fractions I have used so far have been proper fractions.

Improper fractions are those that are formed like a proper fraction with a counting number and a describing number, but the counting number is equal to, or larger than, the describing number. 5 / 4 and 5 / 3 are examples of improper fractions.
Improper fractions can become whole numbers plus fractions, or **mixed numbers**.

4 / 4 is a fraction that really tells us we have a whole object or group.

5 / 3 is a fraction that tells us we have enough parts to make one whole object or group, plus some more parts. 5 / 3 = 1 and 2 / 3.

One last topic related to fractions is this.

Talking about a fraction of something can mean that we really do have something that is already divided into parts, or that it really will be divided into parts.

It can also mean that we only pretend to divide something in parts. We use our minds to make a division so that we can talk about something that we may not wish to divide, or cannot divide.

The children always liked to think of a fraction of their own or someone else's body. Sometimes the topics were gruesome. "3 / 4 of my body is covered with spots", is a tame example.

I'm sure there's more to say about fractions but I've gone on long enough. I do believe that they are something young children can and should learn. It is most important that they understand that a fraction is a special kind of number, and how a fraction gets its value through the relationship between the two numbers in it. With this knowledge they can understand how changes to one number affect the other, and can do so without changing the value of the fraction, only the form.

Part 6, Multiple figure addition and subtraction:

I believe that children do best at math or arithmetic when they are confident that this work is easy for them to do. I tried to start with the simplest examples and bring them to understand that most new work really was built on what they already knew.

We may have been going to start to add and subtract large numbers, but bigger questions were really just a series of little questions they already knew how to do.

What would be new were the steps that had to be taken in order to do the bigger questions.

In my Simple Math booklet, p. 7, I covered only those addition and subtraction questions, or incomplete number sentences, which are basic facts.

I spoke of all of these as being formed by **TWO PARTS in ONE WHOLE**. I used these terms because I felt they were words the children were comfortable with.

Such number sentences were either

PART + PART = WHOLE or **WHOLE - PART = PART** .

Before starting with two figure addition and subtraction questions I would have had the children become familiar with two things, using single figure parts.

One would have been questions having more than two parts, such as

3 + 2 + 8 + 5 = ___ or 17 - 4 - 6 - 2 = ___ .

The purpose of doing this would have been to convince the children that these could be done in small steps that they already knew how to do.

What would be new was the need to hold a number, a partial whole, in their minds as they went from step to step. Also new would be the need to keep track of which numbers had been dealt with.

Such questions allow the introduction of the idea that we can decide which order we wish to do the task in, something they might make use of in later years.

Both examples above are made so sets of 10 can be created (2 + 8 and -4 - 6) . Working with sets of 10 provides an opportunity to reinforce the idea that using place value can simplify our work, and provides practise for the most commonly used basic facts.

That is, seeing 2 + 8 make 10, and 3 + 5 make 8 leads to a simple 10 + 8 = 18. Seeing that - 4, - 6 are the same as - 10 allows an easy 17 - 10 = 7 and 7 - 2 = 5.

<u>Simply doing the examples in the order given requires more difficult thinking for most.</u>

The second thing to introduce was questions written in vertical form. This could have been done while practising the basic facts, and while working with the above questions.

In both cases we first worked with real objects so that the children could see for themselves that:

1. The larger questions were made of a number of small questions and steps.

2. The horizontal or vertical arrangement of a task made no difference to the answer.

3. The task of addition was to put parts together to make the <u>final whole.</u>

4. The task of subtraction was to take a part, or parts, from the whole to leave a <u>final part</u>.

Point 4. above is one area where some children fall down in doing more complex multiple figure subtraction. **They may try to take a whole from a part** when they see that the part is larger than the number in the same place in the whole, as 3 - 5 in

 43
<u>-25</u>

When we were to begin this multiple figure work, my class sat in a semicircle on a carpeted area facing a chalkboard. We had a large sheet of paper, about 16 in. by 24 in., to lay on the carpet, and wax crayons to write on it with. We had blocks and egg cartons cut down to use as holders of sets of 10.

I told the children that the most important part of what we were about to do was not the answer, but how we were to work to find the answer.

I did not want anyone to say the answer.

The reason for this is that some children's eyes glaze over when the answer has been found. They no longer see the need to pay close attention. <u>In fact, some of the more difficult children to teach in the end were those who had learned elsewhere to do the simpler types of questions, but not the more complex ones.</u>

We started with a subtraction question written in vertical form on the chalkboard, and the paper. (Some children have difficulty working with what they see if it's not on the same plane. Some are slow, for example, to copy from the chalkboard onto their exercise books at their desks and make frequent errors.

Put the same work on a paper and set it flat beside them on their desks, or the floor in this case, and they do better.)

I chose subtraction first to make the point that they were going to take a part out of a given whole.

(**NB**: Some people believe in approaching subtraction through finding differences between numbers. This may require there to be two sets of numbers out in view, which can be confusing, and requires some more abstract mental gymnastics.)

The question would have been printed on the chalkboard and the paper. It would have been a two figure whole, take away a one figure part, where **the number of ones in the whole was larger than the number of ones to be taken away:**

4 8
<u>- 5</u>

There would be one group of 48 objects separated into 4 tens and 8 ones on the carpet.

I would point out the tens and ones and review place value briefly.

Very lightly I would draw a vertical line in a different color between the 4 and 8, going down into the answer space, on the chalkboard and on the paper.

I would then ask the children what the question told us to do to the 48, the 4 tens and 8 ones. What I would wanted to draw out was that we were to **take away 5 ones.** If someone said, "Take away 5", I would ask what the 5 were, tens or ones.

Then I would ask one child to go to the blocks and do what was required by the question. **He or she would go to the 8 ones and take 5 ones away.**

We could see 3 single blocks remained, and I would ask where this number should be written in the answer space. This would be the time to stress that this number can't be written just anywhere. It has to be written in the one's place if it is counting ones.

How do we know where the one's place is in the answer space?
The position of the one's, and any other place is decided by the first number written down.

These places extend down in a vertical line. **They form a column for each place**.

Some children run into difficulties simply because they don't keep the places clearly separate so this has to be stressed. At first it is a good idea to use bigger print and spacing, or light lines, on worksheets to make it easy to see the separate places.

Now, going back to the example, I would ask if we were finished after the 3 was written down. "How many think, yes? ... How many think, no?..." If any thought 'yes' I would have someone tell us what we started out to do. Then I would ask if 48 - 5 could be 3.

I would point out that the question told us to take away 5 ones, but 48 has tens and ones. "Were we asked to take away any tens?" ... "No."
Then I would lightly print a 0 in the tens place beside the 5. "Is this how many tens we were to take away?" ... "Yes."

There was a second step. We had to answer 4 tens take away 0 tens.

The answer was 4 tens and 4 was written in the ten's place.

`The full answer was 43, and this would be the first time I would wish the answer to be spoken as forty-three.

NB: If you haven't gone through this for the first time with young children, particularly those who have difficulty with new learning, you are probably thinking this is all too slow and too concerned with obvious little details.

The truth is I have probably forgotten to mention a lot of those little details.

We older people have done questions like this countless times. What we do becomes automatic and effortless. Once automatic it is not easy to see all the places where young children can have difficulty.

Certainly there are easier and quicker ways to do 48 - 5, but **what the children need is to learn a pattern of behavior that will serve to help them solve any of the larger and more complicated questions to come.**

Later on attention can be paid to methods of doing some questions in alternate and quicker ways, after they have mastered this pattern of behavior.

Knowing when to apply other methods requires them to discriminate between questions, something they probably are not ready for yet.

It hasn’t been my intention to wrap this up into neat little lessons. There are so many different classes and situations that teachers have to deal with, things they have little control over. It is my hope you can take what you need from what I have here and adapt it to your situation.

Going back to the multiple figure work, I would want to do several more examples with the children where we were on the floor. After this I would have a printed exercise for them to do at their desks, one that practised the very same type of question, simple subtraction, where the one figure number taken away was smaller than the number in the one's place of the two figure number.

I would stress that although I might have given them ten questions, there really was twenty to answer. They had to learn that 48 - 5 is to be done in two steps. First, 8 - 5, and second, 4 tens - 0 tens.

I would also ensure that everyone was writing the answers in the correct places, that is the 3 directly under the 8 and 5, and the 4 directly under the 4 in 48.
This required some children to shrink the size of the numbers they had been used to making.

On the next possible day the class would gather as before with the same materials.
This time we would do an addition question of the type where a two figure number is added to a one figure number.

The whole created in the one’s column will be less than 10.

4 3
+ 5

There would be two sets of objects on the carpet, one with 43 grouped in tens and ones, and the other with 5 ones. The question would be on the chalkboard, and a large sheet of paper on the floor.

I would ask the children to tell me what we were supposed to do. Likely someone would say we were to add the 5 to the 43. I would leave it at that.

Then I would ask one child to do the work. I’m quite sure he or she would simply put the 5 in with the 3. (Remember, the 40 blocks were in cut off egg cartons.)

This would give me the opportunity to ask which two numbers were really added. Most children by this time can see that the 4 tens remains untouched.

What we did was simply 3 + 5.
Then we would print the answer, 8, in the one's column.

"Are we finished?" No, not until we've done the second question.
I would ask the children to tell me what was being added to the 4 tens on the floor. Obviously, nothing. The second question then was 4 tens plus 0 tens.
The answer, 4, was written in the tens column.

After this we would proceed just as we had before with the subtraction question, 48 - 3.
We would do more examples together and then practise exercises would be done at the desks. Again, I would have watched to see that numbers were being printed in their proper places.

Why start with 48 - 5 and 43 + 5 ?
This forces children to start with the one's place question.

Some children will quite naturally try to start with the place on the left.
Sometimes this will make no difference.
<u>But, what we are trying to do is to establish good habits that will help the children do any multiple figure addition or subtraction question.</u>
Later they can learn to stray from these habits if they have a good reason.

Next, I chose to go to questions where **the number taken away or added on had no ones.**

** 48**
<u>- 20</u>

This time I would ask what is to be taken away from the 8 ones, and from the 4 tens.

We would then do 8-0, and write the answer, 8, in the one's column.

I would explain that we started with the one's place question because we needed to develop a good habit, one that would help us later.
Right now the way we worked was more important than seeing or getting the answer.

Then I would ask a child to go to the 4 tens and 8 ones of blocks and do what the ten's place part of the question asked us to do. From the 4 tens the child would take 2 tens.
The 2 tens left behind were recorded with a 2 in the ten's column.

More of these would be done before practising them independently. Again, I would pay close attention to answers to see the numbers were recorded in the proper places.

A similar type of addition question would be worked with next.

23
+ 20

This one would be dealt with just like the others.

The next step was to put non zero numbers in both ten's and one's places of the number being taken away or added on. In subtraction, the number of ones in the whole, the number being taken from, was more than the one's being taken away. In addition the number of ones or tens being added formed a whole less than 10.
36 -24 = ___ , and 52 + 33 = ___ , written in vertical form, would be examples to do.

We would have proceeded with these, **separately**, just as before. We would use real objects, grouped in tens and ones, start with the one's place question, and record the answers. Then we would practise similar examples.

By this time I would start to say, "What is the **complete** answer?" when we finished.
I would start to remind the children that although we might be doing 6 - 4 = 2 ones, and 3 tens - 2 tens = 1 ten, what we were really doing, in steps, was 36 - 24 = 12 (twelve).

It was time now to start practising some discrimination between types of questions. First we had some discussion about mixed examples of addition and subtraction questions on the chalkboard.

What I wanted to review was the practise of checking the operation sign, the + or -, before starting a question.

Some of the addition questions were of the type where there were zero one's in the part being added to, as 70 + 23 = ___ , in vertical form.
Some subtraction questions might have had the zero in the answer space of the one's place, as 43 - 23 = ___ , in vertical form.
Zero may mean nothing but it is surprising how much trouble it can cause some children. There are times when it is important to put 0 in an answer, and times when we customarily leave it out.

I also needed to remind the children that although the operation sign was only printed once, it applied to the little questions in both places. I have had children make errors by subtracting in the one's column and adding in the ten's.

We spent a few days doing practice exercises with mixed addition and subtraction questions of the type we'd studied. When the children were fairly comfortable doing these we moved towards the more difficult questions.

We started with those addition questions where **the one's place answer will be less than ten, but the ten's place answer will be more than nine**.

6 3
+6 4

We worked with real objects grouped in tens and ones.

When a child pulled the 3 ones and 4 ones, and 6 tens and 6 tens together I asked what the rule was for our place value number system. **1->|<-10**
The 7 ones were fine, but there were 12 tens. What did that mean?
We piled 10 of the ten's groups together and saw that we had 1 hundred, and 2 tens.
What would we write as an answer under the ten's column? 2.
Where would we write the 1 hundred? To the left of the ten's place!
That was where the hundred's place was.

We proceeded as before, but again, in checking practice work I had to look carefully to see that the answers to each step were written in their proper columns.

Now that we knew that sometimes the answer in one addition step can move into the next place to the left, we could start to try the most difficult addition.

But, before going on it was time for a great confidence booster. The whole class would do a 12 place addition question together on the chalkboard. I wrote down:

2 3 6, 4 8 3, 5 9 1, 6 1 8
+3 4 1, 4 1 5, 4 0 5, 1 0 1

Note that I have taken care not to let any of the parts in one place total over nine.

Then we did the same for subtraction. I may have put up another two and allowed a pair of children to try them at the chalkboard. They felt good about this and it was a reminder that doing one big question can just mean doing a lot of little ones.

After this it was time to go back to the blocks, as first described on p. 25, to do questions where **the parts in the one's place form a whole greater than 9 when a one figure number was being added to a two figure number.**

4 7
+ 5

As before, we did this in steps, starting with the blocks. When a child pulled the 7 ones and 5 ones together we knew we had 12 blocks. I wrote 12 down **away from** the question.

Obviously it was a two place number. Could we write it in the one's place? No. We could by now see the 4 tens waiting to come down into the answer, and 412 would not be a reasonable answer.

The 12 tens needed to be grouped. Our number system required it.

I wrote down the old **1->|<-10** to emphasize this.

When grouped, we had 1 ten and 2 ones.

What could we write in the one's column? 2. There couldn't be any more ones. Where should the 1 ten of blocks go? They belonged with the other tens.
Where would we write it? We could write it at the bottom of the ten's column, but there were more tens to be considered, and they had to go there too.

(Here I differ from some teachers.) I taught the children to print the 1 as a tiny number above the ten's column. My feeling is that this crutch can be discarded later, but right now it a very important visual aid. It is especially helpful when it comes to correcting errors.

Now we had to add 1 ten, 4 tens, and 0 tens. We recorded 5 in the ten's column. The full answer was 52.

After doing some more examples I showed the children how we could picture these questions on scrap paper.

We practised putting enough x's down to equal each part of a question.
The x's were grouped by circling 10's, and these picture numbers were arranged with one part above the other, tens above tens and ones above ones.

We circled 10 ones and drew an arrow from that set to the top of the tens.
Then we drew a horizontal line under it all and wrote the number answers at the bottom.

Each child was given some scrap paper, and a set of questions of the type above. They were to draw pictures of the parts being added together if they had difficulty.

I spent a lot of time going around to help, and to check to see the procedures we had practised together were being used correctly. A few children might have had to go back to the blocks.

When the children were comfortable with this type of question we moved on to those addition questions where both parts have two figures, but **only those in the one's place form a whole larger than 9.**

** 5 6**
<u>+2 8</u>

Note: It is important to see that, after grouping, the ten's place is not more than 9.

The lesson for this type of question went as before. The children were given scrap paper to picture the parts they were to add while doing practise questions at their desks.

Next to be learned was the similar question where **both places will form a whole larger than 9, sometimes only after the one's are grouped to form a new ten.**

4 5
+5 6

After going through the same teaching routine with this type, I told the children that they may hear others say they were **"carrying"** when describing the act of forming a set of ten and moving it to the place to the left as one more there.

At this point some of the more advanced children might have enjoyed trying questions with 3 or more figures. If this didn't seem advisable, I demonstrated on the chalkboard how any very large addition question (They would give me one.) could be done using just what we had learned so far.

This was a good time to review how doing large questions can be nothing more than doing a lot of little questions, carefully, and in the correct order.

When the children were comfortable with all the addition questions requiring grouping, it was time to have them learn to **discriminate** between those simple ones that we did first with two little adding questions, and the more complex ones we had just worked on that required grouping as well.

We put up one question of each type on the chalkboard and discussed how we could tell which one would need grouping, or carrying, and which would be simple addition.

We could see that if the one's place parts added up to more than 9 we had a question with an extra step. If the one's place part added up to less than 10 we just had two little addition questions.

Of course, this could apply to the ten's place as well, but at this stage with only two figure numbers this was not really important.

The children then did a number of questions at their desks that were of both types. A few children would put a 1 above the ten's place when they had not needed to create a ten. Some would forget to use **1->|<-10** when it was needed.

When they were comfortable with this work we reviewed the simple subtraction questions we had done before and added some of them to the mixed review questions so they had two basic types of addition, and one of subtraction.

After this we got out the usual things and went back together to the carpet. First we studied questions like 10 + 4 = 14 to see where the 10 and 4 went in the answer, 14. Next **we put down a whole set of blocks grouped in tens and ones, with a small number in the one's place, a number less than the ones to be taken away.** Then I wrote the following on the paper and chalkboard:

$$\begin{array}{r} \mathbf{5\,2} \\ \underline{-\ \ \mathbf{4}} \end{array}$$

I asked the class what the question asked us to do with the 52 blocks on the floor. They usually responded that we had to take away 4 ones. It was emphasised that what we could see was the whole, and we were to take part of it away.
Then I asked one child to come forward and do just that. Often a child would move towards the blocks, take a long look at the single blocks, and back away. They were puzzled. But there obviously was enough blocks to take 4 from.

Eventually someone would suggest that we take the 4 from one of the tens.
I would respond that if we did, it wouldn't be a ten anymore.
If it wasn't a group of ten, what would it be? Ones.
Where should these ones be? With the other ones.
Then I took one egg carton ten and dumped the blocks in with the 2.
How many ones did we have then? 12.
Could we take 4 ones out of 12 ones? Yes.
How many ones would be left? 8.
Now we went to the printed figures.
We had to have a way of showing what we had done to give us enough ones so we could take away the 4 ones.

Well, the first thing we had done was to go to the ten's place and take out one of the ten's. I then drew a diagonal line through the 5 ten's and wrote a little 4 above it.

The second thing we had done was to spill the set of tens in with the ones.
How do you show that a single figure number is now ten more?
You can put a 1 in the ten's place to the left of that figure.
I then printed a tiny 1 to the left of the 2. I told the children this was something we were allowed to do to remind ourselves what would have to happen with real things.

Now they could see that the little questions were 12 - 4 in the one's place, and 4 - 0 in the ten's place.

We then did another example much like this one in the same manner.

The third example had the whole as a number of ten's, but no ones. This was a little disturbing for some, but they soon saw that if we proceeded as before we could do the question.

There was just one extra step, **regrouping**.

The children then did some of this type of subtraction at their desks, a two figure number minus a one figure number, where **the one figure was larger than the number of ones in the two figure number.**

On the next day we reviewed what we had done and practised some more.

Then we went back to the blocks on the carpet to try questions with two figures in the whole, and the part to be taken away, and with the **one's place part being larger than the one's place whole, and the ten's place part at least two less than the ten's place whole.**

7 3
<u>-5 9</u>

We went on just as in the last example, except we had to do 6 - 5 in the ten's place instead of 6 - 0.

Again, while the children were doing practise questions it was important to go around to see that they were following the correct procedure, and printing the numbers in their places.

Some children had to have some real objects to work with to be comfortable that what they were doing was correct.

At this point I could mention that some people would say,**"borrowing"**, when they talked about the regrouping that had to be done in these questions.

On the next day, while reviewing the above and using blocks as before, I would put in a question where **the ten's place part was only 1 less than the ten's place whole.**

8 3
<u>-7 5</u>

This type of question bothered some children. They could see the question in the ten's column became 7 - 7, but should they write the 0? Should the answer be 08?

We looked at our number chart and saw 08 in the first row. I asked what the zero told them. They would respond that it meant there were zero tens.

I asked how many tens were there when we just printed 8. They would answer that there was zero.

Then we considered if the usual way of writing a number of ones required a 0 in the ten's place. Obviously it didn't. If this was true we didn't need to print anything under the 7 - 7. Putting a 0 didn't make the answer wrong. It just wasn't needed, so it needn't be put in.

Again, we practised these questions, and then again with some two figure minus one figure questions, all of which required regrouping.

After this **it was time to learn to discriminate between subtraction questions that required regrouping and the simple ones that didn't.**

This is a very important step. Regrouping is a difficult concept for some to deal with. They get so wrapped up in doing it they will regroup when they don't need to.

This was also a great time to review **the need to make a habit of starting with the one's place**. If a question required regrouping, one could get into a real mess by starting with the ten's place.

Back on the carpet, I put out a single set of blocks grouped in tens and ones, say 4**5**.
Then I asked what we would do if a question asked them to take 6 ones away, 7 ones, 8 ones, and 9 ones. In each case we would have to regroup.
But what if we were to take away 5 ones, 4 ones, 3 ones, 2 ones, 1 ones, and 0 ones. In each case we would have a simple subtraction question.

Then the number was changed so an 8 was in the one's place (any number of tens).
The children found that only one amount, 9, to be taken away would cause regrouping. We could see that **regrouping was needed when the amount of ones to be taken away was more than the number of ones in the whole.**

We could then start some mixed subtraction practice, questions requiring regrouping and questions only requiring simple subtraction.
Gradually I began to add some simple addition questions as well, ones that didn't require grouping.

When they were quite comfortable with these it was time to bring in all the types.

Together we made a classification table on the chalkboard. At the top, to the left of a vertical line we put "Addition", and on the right, "Subtraction". Then a horizontal line was added in the middle, giving us four spaces. (I didn't print the following!)

<u>Top left</u>**: Addition when the parts in any place's column do not add up to more than 9. Here we simply add and record the whole at the bottom of this column.**

<u>Bottom left</u>**: Addition where the parts in any place's column add up to more than 9. Here we have to group the whole in that place's column into 10's and add that number of 10's to the top of the place's column to the left. Then we record the remaining ones of that place's column at the bottom of their column.**

<u>Top right</u>: Subtraction where the part to be taken away in any place's column is less than the number in the whole above.
Here we simply subtract and record the part left at the bottom of this column.

<u>Bottom right</u>: Subtraction where the part to be taken away in any place's column is more than the number in the whole above.
Here we must take one from the place to the left in the whole, cross it out, and put a number that is one less. Next we must take that one to the place to the right where it becomes ten. After that we must put a little 1 to the left of the number in that place to show it is ten more. Then we can take the part away from it and record the part that is left at the bottom of this column.

To these we added examples of different (not all) situations within each type, which I shall put in horizontal form to save space. (*See the **example** after p.42.*)

Top left: 45 + 3 = __ , 45 + 20 = ___ , 45 + 23 = ___

Bottom left: 42 + 73 = ___ , 45 + 7 = ___ , 45 + 27 = ___ , 45 + 57 = ___

Top right: 45 - 3 = ___ , 45 - 20 = ___ , 45 - 23 = ___

Bottom right: 45 - 8 = ___ , 40 - 8 = ___ , 45 - 28 = ___ , 45 - 38 = ___

These were all the types of questions we had gone over together. Now we could do practice work that was fully mixed. This is always a difficult task for some children. Again, you may wonder why I have gone into so much detail with this work. What you must remember is that for young children, doing this for the first time, there is a lot of little details. It's all new. I'm sure I haven't gone into all the problems some will face.

Some of the main points I hope you have gathered are:

1. Children must have a good understanding of place value. They must be able to see 48 as a single quantity called forty-eight, and as two separate amounts, 4 tens and 8 ones.

2. Children must understand that the way to do large questions is to break them down into little questions.

3. Children must have a good knowledge of the basic addition and subtraction facts.

4. Children must be confident enough with place value to see that they can change 15 ones into 1 ten and 5 ones, and they can change 4 tens into 3 tens and 10 ones for a special purpose.

5. Children must be able to discriminate between the questions that require no changes before proceeding, and those that do. That is, they must be able to see which ones require the extra grouping (carrying) or regrouping (borrowing) steps.

Once again I'll remind you that I haven't tried to lay out lessons to follow.

Some of the factors and pressures that can affect individual lessons are: children in a class, teachers (yourself and others), principals, schools, timetables, time allocations, curriculums, training, classrooms, texts, workbooks, supplies, climate, weather, painters, lawn mowers, announcements, fire drills, lost teeth, ...

What I set out to do was to give a sense of the <u>many little things that a child can find to be new</u>, and often puzzling; <u>things an adult handles automatically</u>. My purpose was to make you aware of some these, and to tell a little about how I handled them.

I did not describe my lessons in great detail. Much more went on than what I have spoken about here. But much of what I did not say was related to my own particular class, my own interests and knowledge, and other factors at that moment.

There were also many other times of the day when something would come up that could be related to what we had been learning in math, or arithmetic.

I hope I've at least made you think about the topics in this booklet.

Please contact me if you have any comments or suggestions.
My addresses are on p.1.