Gordon's Simple Math

Contents:

Who might find this booklet useful?

Beginning Teachers, Experienced Teachers, Teachers of Teachers,

Parents helping children, Homeschooling Parents,

may appreciate a new way of looking at early arithmetic or math, something they have not found in the textbooks they have been expected to use, or they may be just searching for some ideas to supplement the texts they are happy with.
There is nothing new in this booklet in terms of math or arithmetic.
Basic Math is a set of timeless and universal truths.
What may be new to the reader is the approach to it.

I hope you will find a degree of order in my approach to very basic early math.
I believe an effort should be made to instill some order in young minds.

We do it with many other subjects, but some adults rebel at doing it here.
This makes math far more difficult than it needs to be.
We teach rules of grammar and spelling, and in music we strive for a very disciplined behavior. We don't see these as stifling creativity.

In the following pages I have tried to tie together some basic concepts in a way that will simplify math and ease the efforts of young minds to learn what they need to in order to cope with the modern world.
(If you find you can't understand some point I have tried to make it could be my error.)
(Please mail or e-mail me. My address is on the next page.)

Gordon's Simple Math

Introduction:

This booklet is just intended to be a framework for thinking about the teaching of early math, or arithmetic, whichever you choose to call it.

Mainly it is about adding, subtracting, multiplying and dividing.

It is not written in technical language, and it is not completely correct as to the use of widely accepted terms.

It is not a text to follow and it will not tell you how to teach.

You will find many spaces to fill with your own knowledge because there will be holes in it.

You will have to decide when and how to use what you find here.

The purpose of this little book is to help you provide children with their own simple framework for building number knowledge on, using words they are familiar with, no matter what text or program you may have to use, or wish to use.

I intend to speak often in the first person to constantly remind you that these are simply my opinions, based on experience.

I spent 35 years in Elementary education, some with Grades 4 to 7, most with children from Kindergarten to Grade 3, ages 4 to 9. One year was taken off to earn an M.Ed in Early Childhood Education.

The Game:

Children like to think in terms of what they know. Most know and love games.

I believe I did best when I taught math as a game-like subject.

In many respects it is just like a game. There are definitions to learn and rules to follow. How well you do depends on knowing the language, the rules, and the many facts that make one quicker at the game. And like many games, speed is important, but second to accuracy.

I don't intend to say how a young child's teacher is going to get this game of numbers started; just that the child, and all the other teachers the child will ever have for number work, are depending on this being done well and thoroughly.

Gordon Scott, September, 1997 (modified January, 2005)

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If you have any questions, suggestions, or comments, address an email to **@telus.net** but first put **m.games** before it. (This address is split to attempt to foil programs which gather addresses for sale to junk mailers.)

STARTING UP:
What is **a** number, in common usage, and what is number?

Well, most of the time it is a symbol for a certain quantity.
Sometimes it is the name of a number that is a symbol for a certain quantity.
Sometimes it is just a name.
Sometimes it is a position in an ordered group.
Sometimes the symbol is the same, but the name is different (3, three, third).

We could talk about number and numeral and ordinal and names. The point is that the question is not as simple as it seems, and common usage flits from one thing to another.

Young children have to be taught this from the beginning and reminded from there on. And basic to the main usage of number is the concept of one, or the unit.

They have to know what game the speaker is playing with number.

If they aren't sure of all this, then is it any wonder their minds can go astray, especially if they aren't brilliant and mathematically gifted?

Consider these questions:
What's on the back of that player's shirt? How many cookies do you want?
How many is 5? What do you say when you see this shape? 5
Who is fifth in line? Is this the third time you've used that excuse?
Are you number 8 in line? Could you count to 20 for me?
What time is it? How long is it until lunch? When is my birthday?

All are about number in the most general sense.

Much of this learning may be started well before a child gets to school.

Most teachers do not see these early beginnings, no matter when they start, as having the importance or status of calculus, for example.

But, if the many uses of number are not clear, and tied to the main concept of number, then a child can be expected to have difficulty learning the basics of arithmetic or math.

Before going on, please consider introducing the word **'equal'** very early, and keep it up as you go on with number work.

Equality is a very important concept.

<u>What is counting like?</u>

It can be a little song sung with meaningless sounds.
It can put things or people in order.
It can be a way of finding the amount of things or people in a group.
It can skip numbers if you count by equal groups rather than by ones.
It has a direction and can be done backwards and forwards.

Counting should have a purpose, but it means more to the young child.

It can grow into a way of developing a sense of the relative size of numbers when the real quantities cannot be seen and felt.

<u>A lot of counting should be done</u>.

At first the main concern may be to focus on the joining of number name and object (two, and two cookies in the hand), number name and numeral (spoken two, and symbol 2), or the one-to-one relationship of object-number-numeral.

Later this becomes more abstract as larger numbers are dealt with.

Early on, <u>the system of naming numbers</u> should be brought in and one should not underestimate the difficulty the names between 10 and 20 can cause. My kindergarten classes enjoyed calling these the “terrible teens”, even though they weren’t all “teens”.

From eleven to nineteen, the English naming system does not have the simplicity found from 20 on, and it does not parallel the simplicity of the written numbers.

Still, it provides a useful contrast that can help children see how numbers are named from 20 on.

Again, my Kindergarten children enjoyed thinking about what counting would have sounded like if we had a completely consistent system: “tenty”, “tenty-one (11), tenty-two, tenty-three, ...”, or better still, “onety(10), onety-one, onety-two, onety-three, ...”

As children learn to count beyond 100, they must continue to develop the idea of a naming system, or reading system.

It is important that children see how easy it is to read large numbers. Once they get past 19 there is relatively little that is new to learn, if they can see how we keep using old names in new ways, with very few new names.

In particular, it is useful for them to see that when we learn to read 387, we can also read 387,387,387 with only two new words needed. Children enjoy doing such things, and doing so gives them <u>a very important feeling of confidence</u>.

It is essential to <u>emphasize</u> this<u> simplicity as often as possible</u>. Many children need to be helped to see how easy numbers really are to work with. They have been told often that Math is hard and their mother, or father, or brother, or sister had trouble with it, or they are just naturally inclined to worry about what is to come.

There will be plenty of times to challenge the more able, without discouraging those unsure of their ability.

Relative value, and place value:

In terms of relative position, and size, one may later start to emphasize questions such as:

What comes before 76?   What comes after 76?   What is between 47 and 49?

Those adults not in on the early stages of counting should note that there are some big steps for children to make in answering: What comes after 49? What is before 60?

In my classes we were counting every day. This was the most regular part of it:

There was a solid paper strip from a cash register roll stapled high up on the wall along the front and one side of the room. On it there were all the numbers from 0 to 203... . Every day we moved a stiff paper marker along this strip to show how many days we had been in school.

Of course we used this strip for other counting as well. We could easily count by tens as every 'tens' number was underlined.

As experience was gained we also used it for varied skip counting of the type you need to count your change. That is, we may have counted the days by tens, and then ones; or by tens, then fives, then ones. (Five and ten places were marked with color.)

Then there was a section of the chalkboard which had three vertical rows of the numerals 0 to 9 printed on it, counting upwards. We had a steel chalkboard and used markers on magnets to indicate the numeral for the day in each of the hundreds, tens, and ones places.

This way the children could sense the relative value of each place, how the ones climbed from 0 up to 9 before the tens could move up one place, and the long wait for the hundreds place to move up one.

To show how the numerals are used over and over, we also had a small chart with three hooks, one for each place value position, and on each hook hung a set of cards from 0 to 9. We changed the cards daily to indicate the days we had been in school.

(We also kept a calendar with hooks for the numbers to be hung on daily.)

What is place value?

Place value came in with counting, but it also needs to be looked at by itself.

We normally use a Base Ten Place Value Number System. We represent numbers by using our numerals 0,1,2,3,4,5,6,7,8, and 9 over and over again in a very special way. Put simply, it means that every time we get a group of ten, we can call it a new kind of 1.

This contrasts with what could be called an Additive-Subtractive System as in the Roman Numeral System where the numerals "I, V, X,L,C,D,M" are used in an entirely different way.

With Roman Numerals if you want to write thirty, you use the X symbol three times: XXX, or ten+ten+ten. In our usual way, we use the symbol 3 and 0 in a special order: 30, or 3 sets of ten and 0 ones.

With Roman Numerals our 29 , or 2 sets of ten and 9 ones, is written as XXIX, or ten+ten+ten-one. It is a very different system with specific rules. I,X,C,M can all be added three times, and can be taken away from either of the next two larger symbols once. V,L,D can each be used once and can never be taken away.

With Roman Numerals there is no need for a 0. <u>With our system the zero symbol is very important, and is the only symbol with two functions. First it is used to show the amount is zero, and second it is used to occupy a place and show that place is there, but empty.</u> In 30, the 3 means three, but that's only part of it. The 0 to the right of it sets the 3 in a special place and for us that place has meaning. A 3 sitting there is read as three tens. In 300, the 3 means three, but the two 0's push it into a new place where we read the three as three hundreds.

<u>This is a good time to reinforce the idea that numbers can have different jobs to do.</u>

Children need to understand this thoroughly. They need to see the real difference between 30 and 03, 57 and 75, 103 and 130 or 301 or...
<u>Real objects help here</u>**.** <u>They need a lot of experience with the idea of place value.</u>

Roman numerals can provide a useful contrast to draw their attention to the workings of place value in our number system, and of course grouping and counting real objects is very valuable.
Note: This need to understand place value comes at a time when many children are prone to reversals, reading and writing 'on' for 'no'.

When we kept track of the days at school each morning in my classes, the number line that ran around the room gave little place value practise, but the two ways we used the numerals 0 to 9 to record the number of days definitely did emphasize place value.

I also made up a simple little set of "number makers" to practise place value, but they had other uses. They were made of stiff durable paper (Bristol Board), about 3 inches by 4 inches. I used a sharp knife to cut slits in the paper, through which I ran two strips of the same material, about .5 in. by 6 in. On these strips I printed the numerals 0 to 9 in a vertical line. The slits were placed so that the strips could be threaded in from the back and out through the back again, leaving just enough of each strip exposed to show one numeral. Another plain strip was stapled to each end of the strips to give more strength, and keep them from being pulled through.

This allowed the children to practise making numbers from 0 to 99, while forcing them to concentrate on place value. We also had little games where each child had a partner, made a number secretly, and then had to decide who had made the larger or smaller one, depending on what I called out after their number was made.

It can take quite a bit to convince some children that 40 means more than 39.

Besides the number line, I also had a large, useful number chart hanging on one wall that helped with counting and place value. It had the numerals from 0 to 99, with the ten's place numerals colored in red and the one's place numerals in black. You will find a small sample of this, and two others, in Part 5 (without the colors).

Some like to start at 1 and go to 100, but 0 to 99 places all the like numerals on the same row. Both are great for finding patterns, such as 00,11,22,33,... or 01,12,23,... (useful to pay attention to on the number line as well). You will find both charts given, and they will have two places throughout (00, 01, 02, ...,09) . This may be confusing for children who haven't started to deal with place value, but ,after they have, I think the 0 on the left forces them to deal with left-right placements, and 0 as a place holder.

Note:

Some teachers like to give emphasis to the ideas of place value by making up counting activities using a different base, eg., forming a group every time you reach 5 and calling that group a new kind of 1. This would be using base 5.

This can be a good way to force adults to see what young children have to learn, but I found it could be confusing for many children and the time spent on it was better used on added attention to place value, base ten.

Games:

Here I'm going to mention Gordon's Games, a set of games I developed which is so named, not for ego's sake, but to try to get away from the problem of choosing a name someone else was using and liable to object to my using it too. I had called them, "Two Up", a more appropriate name, but it had many conflicts with others.

Most of the games in this booklet use a simple set of 20 cards having the ten numerals, 0 to 9, twice each, and are usually played with two children together.

These games deal with simple matching, counting, number order, comparison of value, place value, addition, subtraction, multiplication, division, multiples, factors, and even fractions and decimals if you want to carry them that far.

They are useful in practising skills just covered and keeping up old ones.

For example, my classes enjoyed the games where they picked up two cards each and tried to make the larger number, or smaller number if that was the object of the game. Any misunderstanding of place value showed up quickly here and was soon corrected. As well, the left - right orientation problems of some were helped.

Some class games and language related games are at the end of the booklet.

One of the class games is designed to give practise in addition, subtraction, multiplication, and division facts.

As well it requires children to form their own questions to ask.

These questions start out to be simple, but grow to be to be more complex, in their eyes. That is, 3 + 4 = __ can progress to __ + 4 = 7.

I won't say any more about these games, but they are available from me at my e-mail or snail mail address, and, for free, through my home page, all on p.1.

Addition and Subtraction:

Somewhere in the early introduction of numbers, children are going to be introduced to the idea of combining groups of things to make a larger group; and taking away one group to leave another group. (That is, you can start to learn to add and subtract before you know all about counting and place value.)

This little booklet is mainly concerned with these activities in their simplest form, and as they are recorded with numerals.

In my opinion, good math or arithmetic instruction in the early grades will prompt children to say, " This is so easy! I thought it was going to be hard, but its not!"

In other words, it's the teacher's job to give as many children as possible a good feeling about their ability to handle numbers.

There are times when challenges can be set for the more able students, but in the main, I believe, beginning instruction is best kept as simple and flexible as possible.

For this reason the examples in this book will be of the simplest form. The aim will be to start with something small that can later grow to handle more complex questions.

Wherever possible, rules will be developed and generalizations made that can eliminate as much of the drudgery of memorization as possible, and make what is memorized as useful and flexible as it can be.

I will not deal with word problems, nor will I talk a lot about manipulatives in the hands of the children. These are for the reader to deal with.

Where possible, I will use the simplest words, those already familiar to children, to describe the functions of numbers as used in addition, subtraction, multiplication, and division questions. This may offend some readers who know the proper terms for each number in 68 -37 = 31, but , by ignoring these terms I believe I can better tie addition and subtraction together, and link them to multiplication and division.

I will also deal with addition and subtraction at the same time, but as related opposites. The same will be true for multiplication and division.

What I hope the reader will see is a **simple and flexible structure** to which children can attach the facts they must learn.

It is my aim to make children see that there is little more of real difficulty to learn about adding and subtracting 18 things than there is for 2 things, and beyond 18 it is just a matter of learning methods of using what you already know.

Keep it simple. Tie it together. Build on what they know.

<u>Note</u>: The reader will have to judge when, and how, the following could be taught. Look on these words as general suggestions, no more.

First, its important to have an understanding of 1 as a unit and 0 as a real amount. With that one can start the game of addition and subtraction.

At some point in this game you have to teach these words and their symbols: MINUS or TAKE AWAY (-), PLUS or ADDED TO (+), and EQUALS (=) .

You may feel most comfortable starting with 3 objects. I liked 1in. (2.5 cm.) cubes.

I would start out by speaking of these 3 as being the WHOLE . In other words, for the purpose of this game, 3 is all you've got.

From the 3 you can take away 1. What has been done now is to turn the 3 into two PARTS, a part, 1, that was taken away, and a part, 2, that is left.

<u>Now reverse this and make a real point of the concept of doing the opposite</u>.

Take the 2 that was left and bring back the 1 that was taken away so that 3 appears again. Emphasise the point that the PART that was left, and the PART that was taken away, join to make the WHOLE.

In other words, if you've got 1 thing and a 2 things, it is just the same as having 3.

If you have taught the words and symbols, then you can simplify the above to be
3 - 1 = 2, and 2 + 1 = 3.

<u>Let the children see these objects clearly while you talk about what you're doing.</u>

(I didn't give children their own objects until I was sure they knew the game.)

<u>Notice that I have started with subtraction</u>. I think this best defines the WHOLE and the game you are playing with its PARTS. <u>Subtraction is often misunderstood by children in terms of the relationship between the WHOLE and the PARTS in it.</u>

Now its time to go to the WHOLE ,3, and from it take the PART, 2, leaving the PART,1. Recombine them by starting with the PART,1, that was left and bringing back the PART, 2, that was taken away to make the WHOLE,3, again.

Take note of how doing the opposite brought us back to the start again.

In short: 3 - 2 = 1, and 1 + 2 = 3.

Now compare the sentences to note how the PARTS change places. That is, when 2 is taken away, 1 is left, and when 1 is taken away, 2 is left.

Inside the WHOLE of 3, the PARTS 1 and 2 are inseparable. They are PARTNERS, or RELATED PARTS, <u>inside 3</u>, and they can change places. They can be the number taken away or the number left, the number started with or the number added on, so long as their PARTNER fills the other position, or does the other job.

This is best seen in simplest form: 3 - 1 = 2   2 + 1 = 3
3 - 2 = 1   1 + 2 = 3

Now its time to look for another set of PARTNERS in 3. Start with the WHOLE of three and take away the PART 3. What PART is left? 0!

This is difficult for some children.

Make a point of reminding them that this is like a game and the game **has** to have two PARTS.

Make a definite place for the three things, such as on a piece of paper, or inside string circles.

When the PART 3 is taken away, nothing is left in that place. One purpose of zero,0, is to name the amount, nothing.

It is sensible to say and write, “3 - 3 = 0”.

Now point to the place with the PART 0 in it and bring back the PART 3. You are now back to the start with the WHOLE 3.

In short: 3 - 3 = 0, and 0 + 3 = 3.

Next you can play the game by taking out nothing as a PART, or 0, from the whole. You are left with a PART 3 of course. Put the PART 0 back and you have the WHOLE 3 again.

In short: 3 - 0 = 3, and 3 + 0 = 3.

These sentences can again be compared:

3 - 3 = 0   0 + 3 = 3
3 - 0 = 3   3 + 0 = 3

We have another set of PARTNERS inside the WHOLE of 3, and they can change places or jobs.

As well, we can see that the WHOLE can also be a PART.

WHOLE, PART, and PARTNERS are all words I would have the children using.

By now some readers will be saying this is too easy, too obvious.

It isn't for a number of children at the start. They must know exactly what is going on, especially when the switch is made to number sentences without objects to see.

Besides, the purpose of this approach is to build a framework for thinking about any number.

Remember, working with numbers is like playing a game.

You have to know the rules, the language, and the object of the game.

That is what we are working on.

After dealing with 3, do the same for 4, and then 5.
That done, review all three wholes and their related parts.

List them on the chalkboard or chart paper in an ordered fashion as follows, leaving space to the left for two more sets:

3   4   5
0 | 3   0 | 4   0 | 5
1 | 2   1 | 3   1 | 4
2 | 2   2 | 3

With the whole on the top and the pairs of their related parts below, ask the children to look for the patterns.
(You can count down the left and up the right from 0 to the whole).

It should take little time to develop the sets for a whole of 1, and 2. (fig.1)

1   2   3   4   5
0 | 1   0 | 2   0 | 3   0 | 4   0 | 5
1 | 1   1 | 2   1 | 3   1 | 4
2 | 2   2 | 3

Ask the children if they see something that always happens.

Hopefully they will see that 0 always works with the whole as a pair of parts, or 0 is always the partner of the whole. If not, lead them to it.

Remind them that we can only say this for this game of adding and subtracting.

Ask if the same will be true if we put 0 into two groups. Will 0 be 0's partner?

Now to look at this another way:
Show the children how we can set out the numbers up to 5 in a horizontal line.

0   1   2   3   4   5

Now join the pairs of parts with a curved line rising up, from 0 to a high point between 2 and 3, and then down to 5. Do the same for 1 and 4, and 2 and 3.
You now have a rainbow pattern. Children love to make this, especially with colors.

Take a set of 5 things. Have two places to put them. Start with 0 on the left and 5 on the right. Then move 1 to the left to have 1 and 4, another to the left to have 2 and 3, another to the left to have 3 and 2, another to the left to have 4 and 1, and finally another to the left to have 5 and 0.
Draw attention to how this mirrors the fig.1 pattern and the rainbow pattern.

Do the rainbow pattern for a whole of 4. Note this time that 2 has to work with itself. Then do the whole of 3, and 2 (noting that 1 works with itself), and 1.

(This might be a good time to introduce odd and even numbers if you haven't already. Both patterns show plainly how only even numbers can have a pair of identical parts.)

By now the children should be ready for a general rule. You might like to discuss how memory plays a large part in work involving adding and subtracting, how we can look for things that always happen and call them rules, and how these rules save us memory work.

Ask the children if they see something that always happens in the sets of parts and wholes. If needed, lead them to the zero and its partner, which is always the whole. Develop this as a rule that seems to be true, but will be checked as bigger numbers are investigated.

Go back to fig.1 and erase the pairs of parts that are covered by the rule for zero, or redraw the set. You may wish to print the rule, eg. "0's partner is always the whole." Note how many separate sets of pairs of parts are dropped out, and how 1 has only the rule to remember for it.

Now may be a good time to go back over what the children need to remember about how the pairs of parts in a whole are written into number sentences to describe real situations. I think it is important that this be very clear.

A number sentence is a quick and easy way to do just that; tell what has happened, or is to happen, or might happen.

The movement of objects, such as blocks, while writing number sentences is a way to demonstrate this. I also had children act out the sentences in groups themselves.

It was at this time I brought in a worksheet to keep track of the many number sentences formed by what has been found out so far about 2, 3, 4, and 5.

I used one letter sized sheet , divided into four equal rectangles. In each section there was the same format. An approximately 1.5 in. (4 cm) space across the top with a box in the middle and larger boxes to each side. The middle box was divided in half by a horizontal line, and the lower part of it divided in half by a vertical line.

The whole was to be printed in the top of the middle box, and its two parts being worked with below it in the two smaller boxes. To the left the children drew simple shapes, such as squares or triangles, to match the number of the part on that side, and the same for the part on the right on its side.

Below this was space for four number sentences. Sometimes this could be:

(picture)   WHOLE   (picture)   (picture)   WHOLE   (picture)
PART | PART   PART | PART

____ + ____ = ____	and sometimes:	____ + ____ = ____
____ - ____ = ____		____ + ____ = ____
____ + ____ = ____		____ - ____ = ____
____ - ____ = ____		____ - ____ = ____

Either way, the children could observe a pattern involving the pair of parts and the whole they form.

We learned a form for each type of sentence:

WHOLE - PART = PART   and   PART + PART = WHOLE

This was often placed at the top of worksheets and around the room.

The children used the worksheet to record the sentences for each of the numbers investigated to date. For example, for a whole of 3, in parts of 1 and 2:

(picture)   3   (picture)   (picture)   3   (picture)
1 | 2   1 | 2

1 + 2 = 3	or	1 + 2 = 3
3 - 2 = 1		2 + 1 = 3
2 + 1 = 3		3 - 2 = 1
3 - 1 = 2		3 - 1 = 2

The first worksheet (above left) allowed us to observe the opposite action of addition and subtraction, and for this I always insisted they put the opposites together. The second worksheet let us see the pairs of parts exchanging roles in the sentences.

Note that I started these with addition sentences and could have made two more worksheets by putting subtraction first. These sheets were used often, right up to wholes of 18. (And, with modification, for multiplication and division.)

Using these sheets, the even numbers stood out because, for one set of sentences, two lines were always left blank, and the odd numbers always filled up all the spaces. At least they did when I convinced those who would insist on writing the same sentences twice that this wasn't necessary.

At some time in here you will want to work on addition and subtraction questions, or incomplete addition and subtraction sentences.

Time has to be spent with real objects, and the meaning of the addition and subtraction sentences.

That is, the children have to understand 2 + 3 = __ as being part + part = whole, and that the whole is missing. I don't think it is enough just to say add the 2 and 3.

Paying attention to 'whole - part = part' will prove valuable later when the children apply the same thinking to 2 + __ = 5 or __ - 3 = 2.

Very early on I showed the children that questions are made by simply taking out one of the three numbers in a number sentence.

In fact we worked together to create all the questions one could ask about one set of two parts in a whole. Starting with 5 made from 2 and 3, the children already knew they could make four number sentences, each with three numbers.

This meant each sentence could create three questions:
2 + 3 = 5 becomes 2 + 3 = __ , 2 + __ = 5, __ + 3 = 5.

All are potential stories with missing numbers (word problems), and all have the same form, part + part = whole.

The children also came to see that 4 made from 2 and 2 meant less work, and they were happy to find ways of doing less.

At some point here it became obvious that 5 made from 2 and 3 could yield 4 different number sentences, and 12 different questions (or word problems). 4 made from 2 and 2 could only yield 2 different number sentences, and 6 different questions.

Knowing that 5 is also made from 1 and 4 makes another 12 different questions one could ask, for a total of 24 questions about 5. If you count in those with zero as one part, there are 36 separate questions one could ask about 5 in two parts.

Children can learn that, in their simplest form, subtraction and addition involve two parts in one whole.

They can learn that the same two parts always go together in a certain whole, and there are some things that always happen that can be remembered as rules.

They can learn the form of number sentences, how they are made, and how simple arithmetic questions are created.

Then they can reduce the amount of memory work they have to do, as well as have a much more flexible approach to problem solving.

Just by knowing the zero rule and the fact that 5 is made of 1 and 4, or 2 and 3, a child can answer 36 different questions, or solve 36 different word problems.

Two facts and a rule do all that, if you know how to use them.

Sometimes I would give a sheet of questions out, all involving a few simple facts, just to impress the children with how much can be done with so little.
This makes it all seem easy, and that is important in the beginning.

And when you learn all these simple facts and rules,
they apply to all the basic work a child has to do
in order to add and subtract natural numbers.

Not only that, the flexible thinking they learn applies to the much more complicated work to come, and there is a carry over to multiplication and division.

NEXT: When children are comfortable with the basic ideas, it is time to go on to do the larger numbers up to 10, noting that the amount of memory work is growing, but so are the savings if it is done wisely. (add to fig.1 and call all this fig.2)

6	7	8	9	10
1 | 5	1 | 6	1 | 7	1 | 8	1 | 9
2 | 4	2 | 5	2 | 6	2 | 7	2 | 8
3 | 3	3 | 4	3 | 5	3 | 6	3 | 7
		4 | 4	4 | 5	4 | 6
				5 | 5

Time can be spent looking for patterns in all this. You can also note that the five facts about 10, and the zero rule, will now handle 66 different questions.

You might enjoy working with odd and even numbers as a way of reviewing. Try looking at the pairs of parts this time and draw out the rule that even numbers are made of two like numbers, but odd numbers are made of two unlike numbers.

You might also review equality and the equal sign by reversing the order of the usual number sentence and having:

WHOLE = PART + PART and PART = WHOLE - PART

What kind of real situations would suit these sentences?

Before going on past 10 as a WHOLE, I would have done a lot of work to ensure that the children knew what kind of game we were playing and what the rules were.

They may not have memorised all the facts,
but they were comfortable with the game.

Before going on to larger numbers, the concept of **place value** has to be reviewed, with the promise that this is going to make the work get easier and easier.

The children will have noted that, going from 1 to 10, the amount of facts to remember has grown steadily. They would love to know that they won't have to go beyond 18 with this table of facts (as fig.2), and 18 won't be much more difficult than 2, and a lot easier that 10.

I paid particular attention to the facts for 10. There are many of them, but they are the ones that are going to be most useful in future work.

After we had worked out the facts for 10, I would start with frequent speed tests of 20 questions each.

I kept the facts, as in fig.3, on paper above the front chalkboard so the children could refer to them.

These were informal tests where they only had to fill in the answer to questions printed on the paper.

I sat at the front and as each child finished they brought it to me and I marked down the number of minutes they finished it in. During lunch hour I marked the tests (I just laid them against a score key to make it as quick to mark as possible).

Children who got a perfect score had their name recorded under the minutes from 1 to 5, according to how long they had taken.

Children who made mistakes were given back their papers after lunch hour for corrections to be done.

All this testing only took about 15 minutes out of the day, but I felt it was very important, especially as we moved towards 18.

The tests made the point that these facts had to be memorized if they were ever to be useful.

We discussed the kind of multiple figure work that would be coming.

The children all improved, but it was obvious to all that the fastest were those who did not have to look up to the front and work out the answer. **Memorization helped!**

Eventually I covered the sheets at the front to make the point very clear that they would not always be there, that they had to be put in their minds.

I will always remember Betty in Grade 2, who could do 20 mixed questions, up to amounts of 18, in 30 seconds, and have them all correct. I could never match that!

Before going past 10, remind the children that we made the zero rule up because it would save us work.

Now look at 11 and ask if there is a very obvious pair of parts. It may be helpful to have a visible set of 10 and 1.

(I used egg cartons (dozens) with two sections cut off. This way many objects could be put in the cartons and it was very obvious when a set of ten had been made. Children were also reluctant to break a set of ten unless there were no other choice, and this was helpful later.)

Tell the children to think of place value if they don't see it right away. Draw out the set of 10 and 1 as obvious parts. Do the same for 12, and on up to 18. Develop a rule for one pair of parts in these two figure numbers.
One pair will always be the ten and the other the number in the one's place.

Tell the children we have now found something new to make their work easier.

Next, pay special attention to 12. Remind the children that 0 and 12 are an obvious pair of parts, as 2 and 10 now are.

Then have a very obvious way of showing 12 as a set of 10, and 2 ones. Again I liked my shortened egg carton to show a full set of 10.

Write the following question for all to see: 12 - 1 = __

Ask a child to do what the unfinished sentence says and take out 1. I would be very surprised if the child takes the 1 out of the group of 10. The natural thing to do is to reach for the 2 and take out 1, leaving 1.
Make a real point of this. Write down what was really done: 2 - 1 = 1, and note that the 10 still sits there, untouched, so what remains is that 10, and now only 1 more.

An adult might see it this way (This is not for the children as it is too confusing.):
$$12 - 1 = (10 + 2) - 1 = 10 + (2 - 1) = 10 + 1 = 11$$

Now, all this may seem too simple, but, again it is the process that is important to get started, not the simple fact, 12 - 1 = 11.

Do the same for 11 + 1 = __ . Have a child add 1 object to the set of 10 and 1. The natural thing to do is to add it to the 1, not the 10. Draw attention to the fact that the real activity was 1 + 1 = 2, and the 10 was untouched again.

Try the questions the other way around: 12 - 11 = __ , and 1 + 11 = __.

Note how the real objects still can be kept apart in tens and ones.

Point out that 11 and 1 are a pair of parts that can be easily worked with when you know about place value, and the pairs of parts for 2.

All along you may have been using the rainbow pattern (see p.10). Now do it for all to see, for 10, 11, and 12. Go from 0 to its partner, the whole.

Then start to erase or cross out the pairs of parts that can be handled by the zero rule, place value, and facts already covered. Note that in all cases one part is a two figure number and the other is equal to or less than the number in the one's place of the whole.

Ask the children what the numbers left have in common. (The pairs of parts are all one figure numbers.)

Start a list of pairs of parts under a whole for 11 and 12 as in fig.2 and note that the list of pairs of parts is shrinking after we pass 10.

Now of course you will be doing other work all along to reinforce what is being discovered, and to review and maintain what was learned.

Keep testing the rules, such as that for odd and even numbers, p.14.

As you work through 13, note that now two pairs of parts are handled by place value, and what is already known about 3.

Keep on very gradually, working up to 18 where you will only have 9 and 9 as the pair of really new parts, just like 2 was made of 1 and 1.

Try to find a new pair of parts for 19, one that cannot be handled by place value and facts already discovered. There are none, and the children will be very relieved.

NB:Hopefully, all along you will have been giving them questions such as 14 - 3 = __ , 2 + 15 = __ , 15 - 0 = __ , and 0 + 12 = __ .

They need to know that the number of very new facts is shrinking as they move towards 18 as a whole, but the number of potential questions has continued to grow all along.

Add to fig.2 so that in the end you have fig.3 which ends like this:

11	12	13	14	15	16	17	18
2 | 9	3 | 9	4 | 9	5 | 9	6 | 9	7 | 9	8 | 9	9 | 9
3 | 8	4 | 8	5 | 8	6 | 8	7 | 8	8 | 8		
4 | 7	5 | 7	6 | 7	7 | 7				
5 | 6	6 | 6						

You might want to arrange this differently to show where the other pairs of parts would fit in, but either way there are lots of patterns to look for. Note how each number works with all the others as a part, until it works with 9. Then it disappears from the table.

We also played a class game described in Gordon's Games.

In it children challenged two others to see who would be the first to answer correctly their question about the simple facts, within limits I had set.

It was important to make the question as difficult as possible since if one challenged child was beaten by the other, the questioner was up.
Questions that were too easy resulted in a tie, and the questioner lost the chance to be up.

It was simple to lead the children towards asking questions such as " What number minus 8 equals 7 ?"
I was amazed at how easily these questions were handled, compared to all the years when I had followed the usual textbook or workbook ideas for introducing and developing number facts.

Using the method I have been describing here, children learned to look at a whole of 15 and its parts of 7 and 8 as something like a triangle with a number at each point.

They could start with the whole and go either direction to make subtraction sentences. They could start with either part and head to the other to make addition sentences.
(See p.12, middle, for an easy example.)

To make questions, they simply left out one of the numbers.

We did have workbooks in Grade 1 and 2, and textbooks in Grade 3, and I did as I was expected to do and covered the work in them by choosing pages selectively.

I didn't use all the pages, and I did have a lot of worksheets I had made myself.

A good deal of time was spent on worksheets of the type described on p.12.

When we reached 18 we made a booklet of these sheets to cover all the really basic sentences for wholes of 2 to 18, something they could take home.

As I said on p.1, it has not been my intention to tell you what to do each day.

I hope you will be able to see some ideas you can use at appropriate times.

If you are using this as a home schooling guide you will have to supplement it with other texts and workbooks.

I have left out many areas of learning that come under the heading of Primary Arithmetic, such as word problems, telling time, and measurement.

Multiplication and Division:

This work was gradually introduced as we continued to work with addition and subtraction.

I pointed out that the even numbers could be made with two equal groups, and this was what multiplication and division were about, equal groups, sets, or parts. It was made clear that this was a new, **but similar**, kind of game with different rules.

Notice that I have switched from using ‘parts’ to using ‘groups’ most often.

Addition and subtraction could work with any two groups or parts, but multiplication and division were games we played with equal groups or parts. We now needed to know the size of just one group, and the number of groups in the whole.

As before I tried to keep the work simple and consistent in the beginning.

We might start by making up sentences about the even numbers from 6 to 18, using what we already knew. Blocks were used to demonstrate the equal groups.

The purpose was to establish the language of multiplication and division, the meaning of 18 -:- 9 = 2 and 2 x 9 = 18. (-:- will have to do for a division sign)

The language I used for the above was ‘18 made into groups of 9 gives us 2 groups’, and ‘2 groups of 9 make 18’. I used this and similar words for some time until they were used to looking at the -:- and x symbols and thinking of those ideas.

Much later I introduced the words ‘divided by’ and ‘times’.

NOTE:

You may wish to reverse the sentences below and the language used because you disagree with the order, and that is fine as long as you are consistent.

For me, I taught the children that the sentences were of this form because I liked the simplicity of the words for the symbols x (groups of) and -:- (made into groups of):

WHOLE -:- (made into groups of) SIZE OF GROUPS = NUMBER OF GROUPS

NUMBER OF GROUPS x (groups of) SIZE OF GROUPS = WHOLE

When this language was becoming familiar we started looking at each whole in turn, from 2 to 6.

It soon became clear to all that because this game had to have equal groups we were not going to have as much to say about certain numbers, and certainly not as many pairs of numbers as we had for addition and subtraction.

To promote a clear break from addition and subtraction sentences I started early, but not immediately, to introduce some new words: factors, and multiples. These made a real distinction between the pairs of numbers that worked together to make a whole in addition and subtraction, which were simply parts of a whole.

We started finding the important facts with a whole of 2 blocks. The children would demonstrate that it could be made in 2 equal groups with 1 block in each.

A real point had to be made here that we were counting different kinds of things. 2 could tell how many blocks there were, and it could tell how many groups there were. 1 could count how many blocks there was in each group.

This was something that had to be constantly stressed.

Then we did the same for a whole of 3 blocks.
We recorded these as 2 -:- 1 = 2 and 3 -:- 1 = 3 .
Next, we put the groups together to make the original wholes and recorded them as 2 x 1 = 2 and 3 x 1 = 3 .

At this point we reviewed what equal meant, how it told us that there would not be a group that was different in number.

Then I introduced the thought that we could have just one group, and there would not be a group that was different, so this could still fit our game of multiplication and division. (Sometimes a child would suggest this before the subject had to be raised.)

With this in mind, we looked at 1 group of 2 and 1 group of 3, and recorded them. Now we had 2 -:-2 = 1 , and 3 -:- 3 = 1.

At this point we had (using ‘groups of’ for x and ‘made into groups of ’ for -:-) :

2 -:- 1 = 2   3 -:- 1 = 3   or   2 -:- 1 = 2   3 -:- 1 = 3
2 x 1 = 2   3 x 1 = 3   2 -:- 2 = 1   3 -:- 3 = 1

2 -:- 2 = 1   3 -:- 3 = 1   2 x 1 = 2   3 x 1 = 3
1 x 2 = 2   1 x 3 = 3   1 x 2 = 2   1 x 3 = 3

After this we went back to 1 and recorded it as 1 -:- 1 = 1 and 1 x 1 = 1 .

Next came 4, and then 5 objects. I won’t write out the sentences for them, but we started to record what we had learned just as we had for addition and subtraction:

1	2	3	4	5
1 | 1	1 | 2	1 | 3	1 | 4	1 | 5
	2 | 1	3 | 1	4 | 1	5 | 1
			2 | 2	

This time we had the numbers underneath the whole as the number in a group on the left, and the number of groups on the right. It was obvious that these numbers exchanged places or jobs in the multiplication and division operations.

Now was the time to introduce the word, ‘factors’.

This made a good break from addition and subtraction.

The numbers under the whole became related factors of the whole, and a very different set of partners.

It was understood that factors could be the size of the equal groups, or the number of equal groups, and sometimes both. Factors could change jobs.

Then, we reviewed the zero rule for addition and subtraction:

If 0 is one part of a whole, then the whole is the other part.

It was easy then to discover a similar rule for working with a factor of 1in multiplication and division:

If 1 is one factor of a whole, then its related factor is the whole.

We could see here that our work was getting easier, just as in addition and subtraction.

Next we had to discover what happened when we tried to work with 0 in multiplication and division.

Could we make 2 into 0 groups? It didn’t make sense.

Could we make 2 groups of 0? Sure, but we would then still have 0.

After trying this with a few numbers it became obvious that if zero was a factor, the whole had to be zero, and any number could be the related factor.

Multiplication and division were not as difficult as they seemed at first.

We did not have to worry about factors of 0, 1, and the whole itself.

Out of all the wholes from 0 to 5, only one had something special to remember, 4.

Then we worked on 6. We checked its behaviour with groups of 1, and 1 group. They followed the rule for 1.

We made 6 into groups of 2, and got 3 groups. We made it into groups of 3, and got 2 groups.

Working with objects, it was obvious we couldn’t use 4 or 5 as factors, just as 3 couldn’t be a factor of 4. Many numbers inside a whole could be ignored.

We could also see that when the related factors were different they still could trade jobs, or positions in the number sentences.

We could write what we had to memorize as:

0 - rule	1, 2 , 3 , - rule	4	5 -rule	6
		2 | 2		2 | 3

Next, we tried 7 things and found 7 also followed the rule for 1 as a factor, and there was nothing else important to be remembered about it for multiplication and division.

It was time to introduce some new and useful words.
We reviewed how 1 was known as a unit, and still would be.
We then looked at 2, 3, 5, and 7.
They all worked the same way in **this game of equal groups.**
The children were told that any numbers like this are called prime numbers.

Prime numbers have no important factors, just 1 and themselves.

4 and 6 were not prime numbers. They were called **composite numbers**.

Composite numbers have factors besides 1 and themselves.

Composite numbers were the ones that were going to give us different facts to remember. Composite numbers could be broken into special equal parts.

We worked on 8, and then 9 objects. With this we could sum up our findings as:

(fig.4)	4	6	8	9
	2 | 2	2 | 3	2 | 4	3 | 3

These facts were displayed in the classroom as before.

With this we could go back to the worksheet frame that was used for addition and subtraction, on p.12, substitute x for + and -:- for - , and write all the separate important sentences for 4, 6, 8, and 9 on one sheet with space left over.

(The picture boxes were used to draw the two groupings (one for 4 and 9) the factors allowed. For example, with a whole of 6, it was [oo] [oo] [oo] and [ooo] [ooo].)

Try that with addition and subtraction! This makes multiplication and division seem so easy, and again, it is important that children think the work is easy. Another way of saying this is they have confidence in their ability to handle the work to come.

Now it was time to work with questions. The formation of questions was presented just as for addition and subtraction; a number was missing from a normal multiplication or division sentence. Having done similar work before, it was easy to see that 6 questions each could be made up about 4 and 9, and there were 12 each for 6 and 8.

Expanding this to 10 was so easy since our hands put the important factors right in front of us. Hold up your two hands and you have 2 groups of 5. Put your hands together so that just the fingertips touch and you will see 5 groups of 2 .

Our hands made this a good point to review the ideas taken so far, and to do a lot of simple multiplication and division questions.

Speed tests to stimulate the memorization of the important facts, as described on p.17 were started now.

Questions involving 1 and 0 were also given. Asking what 347 -:- 1 equals, or 5923 x 0 is, reminded the children that these rules were for all time.

Geometric Shapes:

As we paused at 10, it was time to add a little interest to the review by noting some special arrangements of objects placed in equal groups and fitted together so that the resulting shape had no holes or pieces sticking out.

Blocks (cubes) were very useful in showing that 2 groups of 2, placed together in one way, made a shape we call a square. We called 4 a square number.

6 could form an ordinary rectangle and 9 could form another square. We called 4 the square of 2, and 9 the square of 3.

8 blocks would form an ordinary rectangle but, since it was made of 2 squares of 2, if one square of 2 was set on top of the other we could see the three dimensional shape we call a cube. We called 8 the cube of 2.

All the prime numbers were found to only form long rectangles, 1 block wide, and any number could do this.

The ordinary rectangles could be looked at two ways to show that :
2 x 3 = 6 and 3 x 2 = 6 , 2 x 4 = 8 and 4 x 2 = 8 , 2 x 5 = 10 and 5 x 2 = 10

With this little bit of interest added we now went on to investigate numbers to 18.

11 was prime, but 12 was a most interesting whole. It had two sets of related factors, 2 and 6, and 3 and 4. You could make 8 sentences about 12, and ask 24 questions, without using 1 or 12.

12 formed two shapes, both rectangles, and this gave us the chance to talk about how easy 12 objects are to package up, compared to 10 which only forms one rectangular shape.

We talked about how 12 was called a dozen and many things were sold by dozens. We noted the relationship between multiplication and division and fractions of numbers when we talked about half a dozen.

This gave us a chance to ask about half of 10 (as plain as a hand in front of our face), 8, 6, and 4. Fractions also talked about equal groups.

You could have half of 2,4,6,8,10, or 12 without cutting up units, but not the others.

We were back to even and odd numbers.

You could have a third of 3, 6, 9, and 12. You could have a quarter of 4,8, and 12. You could have a fifth of 5 and 10, and a sixth of 6 and 12. Obviously, 12 was the number that would divide up in the most ways. It had the most important factors.

Each number was beginning to have a character of its own!

At this point I introduced the word '**multiples**'.

We talked about factors having multiples , and the reverse, just like children have parents and parents have children.

The multiples of 2 were 0,2,4,6,8,10,12... We were back to counting!

The multiples of 3 were 0,3,6,9,12... This was easy.

Multiplication and division were related to counting by set amounts.

Now I asked for predictions. Which numbers to come would be composite? It was not hard for many to see that this would be all the even numbers,except for 2.

Would all the odd numbers be prime? Well, 3, 5, 7, and 11 were, but 9 was composite.

Going on from 12, the children were asked to make a prediction for 13. Most thought correctly that it would be prime. 14 was predicted to be composite because it was even. Most knew by now that numbers having even numbers in the one's place were even themselves.

14 proved to be an ordinary composite that only had two important related factors, 2 and 7.

15 also proved to be an ordinary composite with factors of 3 and 5 to pay attention to, but, like 9, it was special since it was an odd number.

Now, another break to review. We made counting lists such as (could go to 15):

0	1	2	3	4	5	6	7	8	9	10	11	12
0		2		4		6		8		10		12
0			3			6			9			12
0				4				8				12
0					5					10		
0						6						12

We discussed the patterns in this set and how they related to multiplication and division. We counted the steps to get to numbers, such as 10, on different lists.

NB: Take care with this, and similar activities, that the children pay attention to counting the spaces between the numbers. Many naturally want to count what they see in such a list, when it is the movement from one to another they should count.

We also used lines such as the first line above by drawing curved lines to show skip counting by each factor from 2 to 6.

Then we did a variation on the rainbow pattern from p.10.

We set down all the numbers up to the whole, and then used a curved line to join the related factors.

If you put in 0, it joins to no other. 1 always joins with the last number, the whole, and that is all that happens to prime numbers.

Composite numbers had more lines, but they never went over the halfway mark, if they even went that far.

You might at this point look at the expanded fig.4, now fig.5. (could go to 15 now)

4	6	8	9	10	12	14
2 | 2	2 | 3	2 | 4	3 | 3	2 | 5	2 | 6	2 | 7
					3 | 4	

Children should try to be able to look at any of these and work out the sentences that stem from these very basic facts.

For instance, 10 made from related factors of 2 and 5 produce, starting from 2 and moving around the triangle of numbers to the right, 2 x 5 = 10. Starting from 5 and going to the left, 5 x 2 = 10. Starting from 10 and going to the right, 10 -:- 5 = 2 , and to the left, 10 -:- 2 = 5 .

Note that 12 has two such triangles.

I now introduced another short way of recording these facts as multiple and related factors: MULTIPLE : FACTOR(S)

For 12 I would write 12: 2 , 3 , 4 , 6 . For 9 it was 9: 3 .

This led to a chart such as this with patterns to investigate:

4:	2					
6:	2 ,	3				
8:	2,		4			
9:		3				
10:	2,			5		
12:	2 ,	3 ,	4 ,		6	
14:	2 ,					7
15:		3 ,		5		

Please remember that all the time this is going on the children are doing questions relating to the facts. These side trips just allow a chance to review and consolidate the facts in a more interesting way.

Going on, 16 was different. It had three important factors: 2, 4, 8. 4 worked by itself.

Investigating the shape of 16 with blocks, as p.22, we found that it was the square of 4, and it also formed an ordinary rectangle. By stacking blocks it formed two big cubes of 2, stuck together.

17 was prime, and 18 was like 12 with four factors: 2, 3, 6, 9. 18 had two ordinary rectangular shapes and could make two cubes of 3.

At this point we could play another kind of game with the multiples of factors.

We wrote on the chalkboard or chart paper the following:

2 -> 0 , 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , ...

If we had started two figure addition and subtraction we may have already played with 'Casting Out Nines'. (See the last part of this booklet for more about this.)

One of the tricks here is to add the figures in multi-figure numbers until they are reduced to a single figure, something that happens to give the same result as taking out all the nines. In this game, nines are like zeros.

Under the above line of multiples of 2 we could then write:

0, 2, 4, 6, 8, 1 , 3 , 5 , 7 , 0, ... Guess what the next would be.

We went on:

3 -> 0 , 3 , 6 , 9 , 12 , 15 , 18 , ...
0 , 3 , 6 , 0 , 3 , 6 , 0 , ...

4 -> 0 , 4 , 8 , 12 , 16 , ...
0 , 4 , 8 , 3 , 7 , ...

In each one we were finding a new pattern and this gave us another interesting way to review the basic facts.

You will find that each of the single figure factors has a set of multiples that yields a unique pattern when 'Casting Out Nines' is used on it, with 9 itself having the most interesting, and useful, pattern (in my opinion).

From here we went on to 19, which was prime. Then came 20 which had the first two figure factor, 10.

Thinking back to addition and subtraction, could this be an important factor to remember? No, it was obvious, knowing about place value, that 20 was made of 2 groups of 10, and this also meant it had 10 groups of 2. The only factors we really had to remember were 4 and 5.

21 gave us factors of 3 and 7.

22 gave us 2 and 11.

Thinking back again to our use of place value, did we really have to pay attention to 11?

No, we could look at each place in 22 and see a 2, which we already knew meant 2 groups of 1. 22 had 2 groups of 1 ten, and 2 groups of 1.

By working with blocks, grouped in tens, we could see that it was obvious that 22 was the first composite number we could ignore.

From here on we knew we didn't have to remember something about every number. Any two figure factor, and its partner, could be worked with another way.

I will not go over every number up to 81. We went on in much the same way as already described here. The farther we went, the better the children got at predicting which numbers we would have to pay attention to and what their important single figure factors would be.

They were delighted to find that 81 was the end of the multiples that had really important factors, just as 18, the reverse of 81, was the end of WHOLES with important PARTS.

All this time that we were working towards 81 we played the class game described on p.18 and did the speed tests as on p.15.

By the time we reached 81, I was able to give the children a single 8.5 x 11 sheet with all the important multiples and factors, with the square numbers separated from the ordinary composites. This will be one of the sheets I'll add at the end.

Odd Bits:

Since the children will have been dealing with two figure addition and understand working with the tens and ones separately, questions such as 3 x 22 = __ and 44 -:- 2 = __ could be considered for variety.

Calculations of the area of simple rectangles gives a concrete purpose for multiplication. It could be as simple as "How many cubes would there be if they were set out, in a rectangle, 6 cubes on one side and 4 on the other?" You don't really have to get into square inches or centimeters.

"How could 24 blocks be formed into a rectangular shape? Is there another way?" This might be a question for the more able children.

Questions involving the price of objects provide another real life source of multiplication and division situations.

See the next section for ideas about how fractions can be related to multiplication and division practice.

<u>Other thoughts:</u> **Two figure addition and subtraction:**

At this point I will leave the development of multiplication and division to go back to addition and subtraction. By this time it had been carried into work with two figures.

I don't intend to say much about this, except that the work you have done with place value will be important here, as well as a very quick and accurate recall of number facts .

I found the children who were unsure at first really needed visual aids, not just objects to manipulate but little numbers on their paper to remind them what is happening. I know it is messy, and some think the grouping and regrouping (carrying and borrowing) should be done in the head, but that is very difficult for some children at first.

<u>Children who really weren't clear on the structure of WHOLE - PART = PART always seemed to have difficulty with subtraction with regrouping (borrowing).</u>

<u>They would see nothing strange in taking the one's place WHOLE from the one's place PART if the usual way seemed impossible.</u>

I found a great deal of value in simply spending a lot of time learning how to discriminate between the questions that needed grouping and regrouping, and those that didn't. I won't elaborate here, but there are some clear signals.

Lastly, especially for those having difficulty, I found it most helpful to spend a lot of time developing a clear set of steps to take in working out each type of question.

Fractions:

We did not do a lot of work with fractions in the primary grades, but the work we did with equal groups suited the study of simple fractions where they were fractions of a group of things.

By learning that the **bottom number** (<u>denominator</u>) **named** the number of equal groups in a set of things, and the **top number** (<u>numerator</u>) **counted** the number of those groups the fraction was talking about, it was easy to relate this to division and multiplication. Eventually the children could find 3/4 of 12 by first dividing 12 by 4 to find there was 3 in a group, and then multiplying that 3 by 3 to get 9.

This provided another way multiplication and division could be practised.

Fractions and Factors and the Least Common Denominator:

<u>A knowledge of factors can be used to help when learning how to add and subtract fractions with different denominators, something older children are faced with.</u>

a.3/4 + 1/6 = ___ . b.9/10 - 5/12 = ___ . These questions require children to change the fractions to a common denominator so the numerators can be added or subtracted. <u>This can be a very trying experience, but it need not be so bad.</u>

In a., the denominators are 4 and 6. Simple multiplication gets a common denominator of 24. Trial and error can arrive at the least common denominator of 12.

But, knowing that the object is to find a number to multiply both parts of the fraction by to arrive at a common denominator allows us to use our knowledge of factors.

<u>Factors tell us how a number is built in a basic sense, especially **prime factors**.</u>

Note: Read the bottom of the page before you skip this one.

Prime factors are those factors that are prime numbers.
All composite numbers can be represented by a set of prime factors.

In a. above, the denominator 4 is 2 x 2 , and 6 is 2 x 3. A common denominator will contain the 4 (2 x 2) and the 6 (2 x 3). That is why multiplying 3/4 x 6/6 , and 5/6 x 4/4 works, but it is not the simplest way. The denominator 24 contains an extra factor which will later have to be removed to arrive at the simplest form of the answer.

If, in a., you consider the prime factors, you know that the least common denominator will contain 2x2 and 2x3. Multiplying these results in 2x2x2x3, but there is a factor of 2 in there that is not needed. If you see they both have the same prime factor 2, then you know the least common denominator will drop one of the 2's.

The LCD (least common denominator) will be 2x2x3 since that will form the smallest number created by multiplying 4 and multiplying 6. Looking at this you can see that 4 x 3, (2 x 2) x 3), and 2 x 6, 2 x (2 x 3), will form the LCD of 12.

The question becomes 9/12 + 2/12 = __ , something quite manageable.

The answer, 11/12, is already in its simplest form.

In b. above, the denominator 10 is 2 x 5, and 12 is 2 x 2 x 3.

10 needs a 2 and 5. 12 needs two 2's and a 3. There is an extra 2 if you simply multiply and get 2 x 5 x 2 x 2 x 3. We only need two 2's, a 5, and a 3.

The LCD is 5 x 2 x 2 x 3. In it there is 10, (5 x 2) x 2 x 3 , and 12, 5 x (2 x 2 x3) .

You can see that 9/10 has to be multiplied by 6/6 , and 5/12 by 5/5.

The question becomes 54/60 - 25/60 = __ , which looks better than 108/120 - 50/120 = ___ that you would have to do if you just multiplied the denominator numbers to get the common denominator. Besides, the answer, 29/60 is already in its simplest form, whereas 58/120 is not.

Obviously, simplifying fractions is another area where factors can be of use.

Asked to simplify 36/45, a child can divide top and bottom by 3 and get 12/15, and by 3 again and get 4/5.

But a child that can see 36 as 4 x 9, and 45 as 5 x 9 can easily see that dividing by 9 (common factor) is the quickest way of taking out that factor of 9 and getting to 4/5.

Using prime factors only, children can learn to cross out common factors from the numerator and denominator. 36 is 2 x 2 x 3 x 3. 45 is 3 x 3 x 5. Take out the two 3's from each of them and you are left with the simplest form, 4/5, since 4 is 2 x 2.

Why all this is a booklet about primary arithmetic?

New sets of knowledge or skills in Math and Arithmetic are built on top of what has come before. That is why we speak of basics. But really, every step required to do higher math is basic to the final outcome.

I hope you can see that something simple, such as factors and prime numbers, started early, will have an immediate value, and a later use, just as place value does.

Think about it. Would you like to hear about factors and prime numbers for the first time when faced with LCD's, or would you like to have known about them well before?

Prime factors can be used to provide useful practice, and fun. With a set of cubes children can show that all numbers , say from 2 to 20, can be made with prime factors.

2, 3, 2 x 2, 5, 2 x 3, 7, 2 x 2 x 2, 3 x 3, 2 x 5, 11, 2 x 2 x 3, 13,
2 x 7, 3 x 5, 2 x 2 x 2 x 2, 17, 2 x 3 x 3, 19, 2 x 2 x 5.

Prime numbers stand out because they are made of one group of themselves.

Composite numbers have two or more prime factors. Those with three or more prime factors have smaller composites inside their sets. 12, for example, has 2 x 2, and 2 x 3.

Children can spend time picking these out as another way of reviewing and reinforcing number facts.

111 Day:

Many schools celebrate or take note of the 100th day at school, but if some effort is going to be put into it I believe that waiting another 11 school days is worthwhile.

Doing something for the 111th day allows the observation of the relative size of 100, 10, and 1, and it also places emphasis on place value. For the first time you have three 1's, all doing different jobs.

I had thought for years that taking note of the 111th day was of more value and had done so in my class. In my last teaching year the whole primary school switched to a 111th day celebration. We carried out similar activities to the 100th day, but paid more attention to the size of 1 hundred compared to 1 ten, and to 1 one.

We had many activities in the gym and after that we had a period of time where half a class would visit another half a class to see what they were doing that was related to 100 and/or place value, and to take part in it.

My class of Grade 2's had enough 1 inch cubes to form a cube of 1000, a square of 100, a line of 10, and 1. This was a great time to review the meaning of large numbers. There was a pattern to see: cube,line of cubes, square of cubes (from right to left).

They also had enough pegs and pegboards to do the same.

We also helped the visitors to make a simple card I had used for years to mark the 111th day. A simple sheet of paper folded twice formed the card.

The front of the card had 111 written in a diagonal line down the page,

1
 1
  1

The children wrote **t 1 o , m 1 y, fam 1 ily** (or mom, or dad,...) on the front.

On the inside left the children made 10 rows of 10 **x**'s.

On the inside right I had already duplicated a diagonal line of 10 large **O**'s. Inside the O's they printed "I love you", which, if you count the spaces, uses all 10 O's.

The children colored and decorated the cards as time allowed and filled in their names on this preprinted section to the lower left of the circles:

"From ________,
111 days at school today"

They enjoyed taking home 100 kisses (x's), 10 hugs (O's), and 1 "I love you".

Casting out Nines:

You can think of this as a simple game we can play with ordinary whole numbers. Once you learn the game it can be interesting, and useful.

All numbers in this game become less than 9.

Any 9 in a number is taken out. Then you only think of the amount that is left.

The effect of this is to separate all numbers into nine families. There will be those that have 0 left, 1 left, 2 left, ..., 8 left. (The 99 chart in Part 5 may be helpful here.)

9 becomes 0 in this game, 18 becomes 0, and 0 becomes 0. They are all in the same family.

21 becomes 3, and so does 12. 15 becomes 6, and so does 51.

101 becomes 2. 79 is a 7. 596 is a 2.

Taking out the nines really involves little or no calculation. You just have to forget about place value.

29 is seen as a 2 and a 9. Throw away the 9 and you are left with 2.

Of course you could subtract 9 three times, or divide by 9, and find the remainder is 2 in either case, but that would take too long for this game.

929,909 can be seen to be a 2 by just forgetting about all the 9's and the 0, or you can do it the hard way.

Most numbers are not so easy, but there is another little trick to use.

247 is in the family of 4 because I can add the 2 and 7 to make a 9, which I cast out, leaving 4.

528 is more difficult. I could start with the 8 and add on the 2 to make 10, which is a 1 in this game. Then I could add the 1 and 5 to make 6.

Or I could have added all three numbers to get 15, which reduces to 6.

How can this work? Well, the 5 is really 500. 9 is 10 - 1. In 500 there are fifty 10's, but to find the 9's we have to remember the fifty -1's. Fifty 9's is only 450, leaving 50 to take 9's out of. There are five 9's in 50, with 5 left over. Altogether, after taking out fifty-five 9's there remains the same 5 we see in the hundred's place of 528.

That 5 in 528 really was the remainder after taking out all the 9's. You can take this farther if you are curious, or doubtful. It really is the same for the 2 and the 8.

What works is to add the figures as single place numbers, add again if the total isn't a single place number, and keeping casting out any 9's you can see.

Example: 35928 is to be done. Look only at the 3, 5, 3, and 8. Add any or all of these together. If the answer is a two place number, add the numbers in those places. Do this until all are added in and only a one place number remains.

eg. 35928: 3 + 5 = 8, 8 + 2 = 10 and 1+ 0 = 1, 1 + 8 = 9, so 35928 reduces to 0.
or 3 + 5 = 8, 2 + 8 = 10, 8 + 10 = 18, 1 + 8 = 9, so we still see 35928 is in the 0 family.

What's the use of this?

On p.27 we used Casting out Nines to find patterns in the multiples of numbers.

Its use here was to provide an interesting review, and, in the case of the multiples of 9, a way of checking the product of a question.

I'll leave it to you to work that one out.

You can use this game as a rough check of the sum (answer) of large addition questions:

2586 + 3498 = 5214?

Cast out 9's in the two parts added as if they were one big number, 25863498. There's an obvious 9, 5 + 4, and 6 + 3 to throw out, leaving 2 + 8 + 8 which reduces to 0.

Cast out 9's in the whole, 5214. The 5 + 4 goes, leaving 2 + 1, or 3.

The addition is wrong!

If both were 0, or 3, the addition would likely be correct.

The proper sum is 6084. 6 + 8 + 4 = 18, which reduces to 0.

NB: It is possible to make an addition error that would give a 0 result. That is why this game is good at finding errors, but not guaranteeing correct answers.

You can check subtraction by treating it in reverse as a addition question. Again, cast out all 9's in the two parts and compare the result with that for the whole.

54.7 - 43.8 = 10.9 ? 547 reduces to 7. 438 and 109 reduce to 7. The computation has a chance of being correct. Notice that decimals have no effect on this checking game.

You can check large multiplication questions by considering them as repeated additions.

523 x 314 = 164,222 ? Cast out 9's in each of the factors, 523 and 314.

523 reduces to 1. 314 reduces to 8. Now multiply the two results, 1 x 8, to get 8.

Now cast out 9's in the multiple, or product, 164,222, and it reduces to 8.

The computation could be correct.

If it were wrong you might use casting out 9's to determine if the error was in your addition, or your multiplication, saving some time.

You can check large division questions, but remainders make this tricky.

Multiply the results from the two parts, and add on the result from any remainder.

This should reduce to the result for the whole. Try one if you like.

The End

*** Corrections are relatively easy for me to make.***

My wife and I have tried to find all the obvious ones, but if you find any errors, significant ommissions, sections that are unclear, or you have some suggestions for me, please let me know. You'll find my addresses on p.1.

A brief description of the worksheet, fact sheets, and number charts:

The flexible worksheet was used with addition and subtraction, and with multiplication and division, to record related number sentences.

No symbols have been put in to allow you to add your own as you wish.

They could be ___ + ___ = ___ , or ___ - ___ = ___ , or ___= ___ - ___ ,

or ___ x ___ = ___ , or ___ -:- ___ = ___ , ...

I would advise you to stick with a consistent pattern on each sheet you use, and to have a purpose for the pattern that you have made the children aware of.

For example, if you wanted them to notice the factors change jobs in the sentences you would have:

___ x ___ = ___
___ x ___ = ___

___-:-___ = ___
___-:-___ = ___

The + - fact sheet with arrows joining pairs of parts and wholes demonstrates the different basic facts children have to memorize.

The + - fact sheet with boxed pairs of parts and wholes has the form of addition and subtraction number sentences I used, and the rules that simplify the memory work.

The x -:- fact sheet with wholes to 12 has the form of multiplication and division number sentences I used and was given out early in this work.

The BASIC x -:- FACTS sheet has the 31 numbers children have to learn basic facts for. Note that 16 and 36 appear in two places, as square numbers and ordinary composite numbers.

The "36 Sets of BASIC Multiplication and Divisions Facts" sheet shows the basic multiples and factors, as well as where the prime numbers are, and the composite numbers having no pairs of factors where both are less than 9.

The "Ninety-Nine Chart" and the "Hundred Chart" are much the same, but I liked the former better. 0 is where we normally count from, and its presence puts emphasis on the steps between numbers, rather than just the numerals themselves.

I've seen many children make counting mistakes because they were counting numerals rather than the moves from one to the other.

The Nines Chart was not used with my children. I made it up to show the families of numbers all reduce to in 'Casting Out Nines'. The top number in each column is the number each below it will reduce to when all the 9's are taken out.