

I HOPE THIS WILL GIVE YOU  
A CLEARER EXPLANATION OF THE TERMS I HAVE  
USED IN MY BOOKLETS AND ON THIS WEBSITE,  
AND ALLOW YOU TO SEE  
JUST HOW SIMPLE AND CONNECTED  
THESE IDEAS SHOULD BE FOR CHILDREN.  
THE ALGEBRA LIKE SENTENCES AT THE TOP OF  
MY HOME PAGE ARE SIMPLY A QUICK WAY TO SHOW  
WHAT CHILDREN  
CAN INTERNALIZE WHEN LEARNING THE FACTS  
THEY MUST KNOW.

I BELIEVE THE METHOD OF LEARNING  
DESCRIBED HERE TO BE A QUICKER WAY TO LEARN  
THOSE FACTS,  
ONE THAT HAS LONG TERM BENEFITS.

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PLEASE TRY TO READ ALL THE PAGES IN ORDER.

PLEASE EMAIL ME IF YOU FIND ANY ERRORS,  
PLACES THAT DO NOT MAKE SENSE,  
OR PLACES THAT ARE NOT EASY TO READ.

I HAVE BEEN ABLE TO DO THIS WITH A NEW MAC PROGRAM.  
CORRECTIONS SHOULD BE EASY.

# THE SYMBOLS, -, +, x, ÷, AND = ARE USED HERE.

WE CAN NAME THEM, MINUS, PLUS, TIMES, DIVIDED BY, AND EQUAL.  
THEY ARE USED IN NUMBER SENTENCES ( $5-3=2$ ), WHICH ARE SIMPLY  
VERY SHORT WAYS TO TELL WHAT IS HAPPENING WITH NUMBERS.

- IS USED WHERE YOU WANT TO SHOW THAT A NUMBER ON ITS RIGHT  
IS BEING TAKEN AWAY FROM THE NUMBER ON ITS LEFT.  $5-3=2$   
(5 TAKE AWAY 3 LEAVES 2)

+ IS USED TO SHOW THAT THE TWO NUMBERS  
BESIDE IT ARE BEING PUT TOGETHER.  $3+2=5$

x IS USED TO SHOW A NUMBER OF GROUPS ARE BEING PUT TOGETHER.  
 $3 \times 2 = 6$  (3 GROUPS OF 2 MAKES 6)

÷ IS USE WHEN THE NUMBER ON THE LEFT IS BEING MADE INTO GROUPS  
OF THE SIZE OF THE NUMBER ON ITS RIGHT.  
 $12 \div 6 = 2$  (12 MADE INTO GROUPS OF 6 MAKES 2 GROUPS)

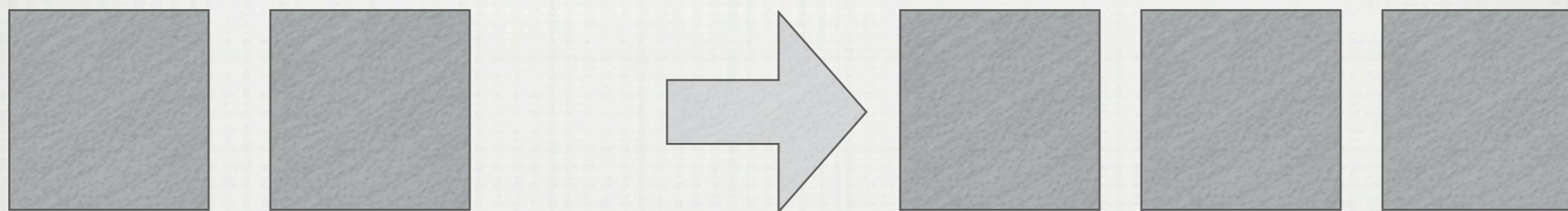
= IS USED TO SHOW THAT WHAT IS TO ITS LEFT IS THE SAME  
NUMBER AS WHAT IS TO ITS RIGHT.

$3=3$ ,  $2+1=3$ .  $2+1$  ENDS UP BEING THE SAME AS 3.

# SUBTRACTION AND ADDITION



WE START OUT WITH THE WHOLE OF 5.



FIVE CAN BE MADE INTO PARTS OF 2 AND 3.

IN THIS EXAMPLE THESE PARTS ARE DESCRIBED BY

THE SIMPLE NUMBER SENTENCES:

$5 - 3 = 2$  AND THE REVERSE,  $2 + 3 = 5$ .

2+3 BRINGS US BACK TO THE WHOLE OF 5.



OR 5 CAN BE MADE INTO PARTS OF 3 AND 2.



THIS ACTION IS DESCRIBED BY THE  
SIMPLE NUMBER SENTENCE,  $5 - 2 = 3$ .  
 $3 + 2 = 5$  BRINGS BACK THE WHOLE OF 5.

THE NUMBER SENTENCES MADE HERE ARE:

$$5-3=2 \text{ AND } 3+2=5 \text{ OR}$$

$$5-2=3 \text{ AND } 2+3=5$$

TO MAKE ALL FOUR SENTENCES (NUMBER FACTS)

WE NEED ONLY THE NUMERALS 2, 3, 5.

CHILDREN CAN LEARN THAT FOR + AND -,

2, 3, 5 ARE ALWAYS TOGETHER.

THEY ARE RELATED.

THERE IS NO NEED TO LEARN

FOUR SEPARATE FACTS.

ALL THAT IS NEEDED IS TO REMEMBER

THE WHOLE, 5 IS MADE FROM THE PARTS, 2 AND 3.

YOU JUST HAVE TO LEARN THE PATTERN

THAT WORKS FOR ALL WHOLES IN TWO PARTS.

THIS SIMPLE EXAMPLE CAN BE REPEATED FOR ANY WHOLE NUMBER MADE IN TWO PARTS.

CHILDREN CAN LEARN THE PATTERN:

$$\text{WHOLE} - \text{PART}_A = \text{PART}_B$$

$$\text{PART}_B + \text{PART}_A = \text{WHOLE}$$

$$\text{WHOLE} - \text{PART}_B = \text{PART}_A$$

$$\text{PART}_A + \text{PART}_B = \text{WHOLE}$$

NOTE: PART<sub>A</sub> AND PART<sub>B</sub> CAN BE EQUAL,

BUT IF THEY ARE, IT'S SIMPLY:

$$\text{WHOLE} - \text{PART} = \text{PART}, \text{PART} + \text{PART} = \text{WHOLE}$$

$$(6 - 3 = 3, \text{ OR } 3 + 3 = 6)$$

THIS PATTERN CAN BE USED FOR ANY NUMBER.

$$54 - 25 = 29, 25 + 29 = 54, 54 - 29 = 25, 29 + 25 = 54$$

$$54 - 27 = 27, 27 + 27 = 54$$

## DIVISION AND MULTIPLICATION

THE OPERATIONS OF SUBTRACTION AND ADDITION  
ARE CARRIED OUT WITH PARTS THAT ARE  
EQUAL OR UNEQUAL.

THE OPERATIONS OF DIVISION AND MULTIPLICATION  
CAN ONLY BE DONE WITH EQUAL PARTS.

FOR YOUNG CHILDREN WE AT FIRST STICK TO  
WHOLES THAT CAN BE MADE INTO, OR WITH,  
EQUAL PARTS THAT ARE WHOLE NUMBERS.  
(NO FRACTIONS OR REMAINDERS.)

JUST LIKE THE FIRST OPERATIONS, THERE IS  
A PATTERN THAT COMES OUT OF THIS,  
A PATTERN MUCH LIKE THAT FOR + AND -.



6 DIVIDED INTO GROUPS OF 2 EQUALS 3 GROUPS.

$$6 \div 2 = 3$$



6 IS THE WHOLE. 2 IS THE SIZE OF EACH GROUP.

3 IS THE NUMBER OF GROUPS.

3 GROUPS OF 2 IN A GROUP EQUALS THE WHOLE 6.

$$3 \times 2 = 6$$

LIKE + (PLUS) AND - (MINUS), OPPOSITE PAIRS OF NUMBER SENTENCES UNDO AND REDO THE WHOLE.



6 DIVIDED INTO GROUPS OF 3 EQUALS 2 GROUPS.

$$6 \div 3 = 2$$



6 IS THE WHOLE . 3 IS THE SIZE OF EACH GROUP .

2 IS THE NUMBER OF GROUPS .

$$2 \times 3 = 6$$

AGAIN, THESE FORM OPPOSITE ACTIONS, BUT THE NUMBER SENTENCES USE THE SAME NUMERALS,

2, 3, 6.

THIS APPROACH TO BASIC FACTS  
ALLOWS CHILDREN TO DEVELOP A FRAMEWORK FOR  
LEARNING, SOMETHING TO TIE FACTS TOGETHER.

IT WORKED FOR ALL  
THE BASIC SUBTRACTION AND ADDITION FACTS.

BECAUSE OF PATTERNS LEARNED THERE  
IT MAKES IT EASIER TO LEARN  
THE BASIC DIVISION AND MULTIPLICATION FACTS.

A NUMBER FOR ONE WHOLE AND  
TWO OTHER NUMBER RELATED TO THAT WHOLE  
FORM 4 SENTENCES. (2 IF THE TWO ARE EQUAL.)

IF YOU LEARN  $9-5=4$ , YOU ALSO KNOW:

$$4+5=9, 9-4=5, 5+4=9$$

BY APPLYING THE LEARNED PATTERN.

IF YOU REMEMBER 8 IS MADE OF TWO 4'S

$$\text{YOU KNOW: } 8-4=4, \text{ AND } 4+4=8.$$

REMEMBER 12 IS MADE OF 6 GROUPS OF 2

AND FROM KNOWING THE PATTERN, YOU KNOW:

$$12 \div 2 = 6, 6 \times 2 = 12, 12 \div 6 = 2, 2 \times 6 = 12$$

REMEMBER 9 HAS 3 THREES

$$\text{AND YOU KNOW: } 9 \div 3 = 3, \text{ AND } 3 \times 3 = 9$$

THE CONCEPT OF RELATED NUMBERS

WITHIN A WHOLE MAKES LEARNING EASIER.

LOOK AT THESE "RAINBOW" PATTERNS FOR + AND -  
WITH A WHOLE OF 7.



OR THIS FOR 8:

8	
0	8
1	7
2	6
3	5
4	4

THESE CAN HELP REINFORCE  
THE RELATED PARTS WITHIN A WHOLE.

BUT BEFORE DOING THIS FOR  $\div$  AND  $\times$   
WE MUST LEARN OTHER TERMS.

# FACTORS, MULTIPLES, COMPOSITE AND PRIME NUMBERS

DIVISION AND MULTIPLICATION REQUIRE  
EQUAL GROUPS.

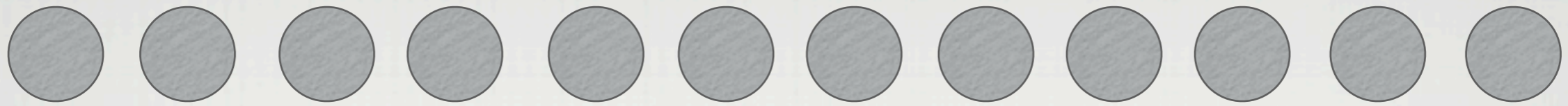
WE NEED TO KNOW THE NUMBER THAT COUNTS  
THE SIZE OF THESE EQUAL GROUPS, AND  
THE NUMBER THAT COUNTS THE EQUAL GROUPS.

OF COURSE WE ALSO NEED TO KNOW THE NUMBER  
THAT COUNTS THE SIZE OF THE WHOLE.

LIKE + AND - NUMBER SENTENCES WE HAVE THREE  
NUMBERS, THE WHOLE, AND THE TWO OTHERS  
UNDERLINED IN RED ABOVE.

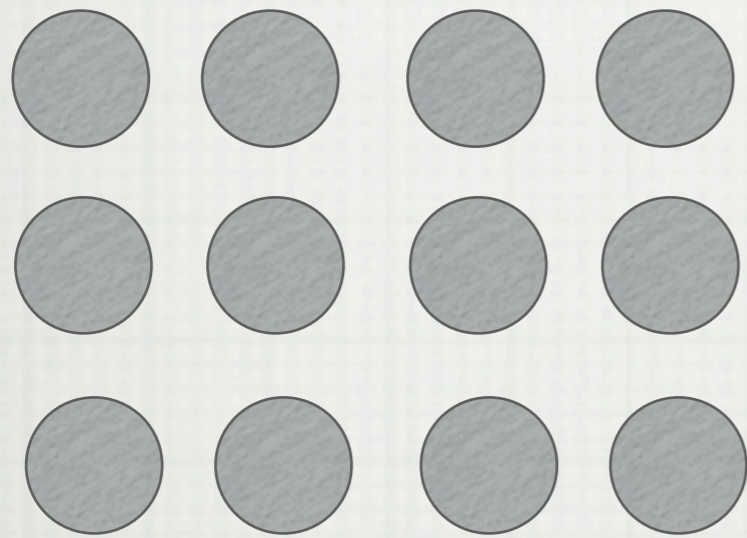
WE CALL THE TWO OTHERS, "FACTORS".

FACTORS WORK TO MAKE A MULTIPLE, THE WHOLE.

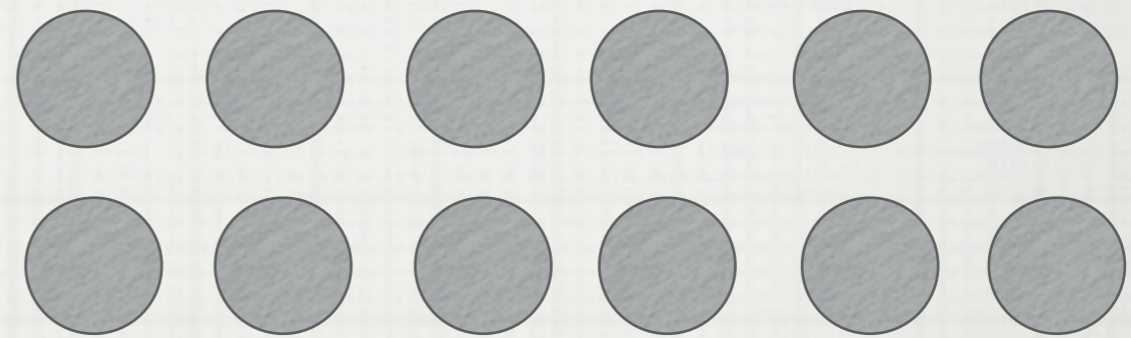


THESE 12 CIRCLES ARE IN ONE GROUP,  
OR GROUPS OF ONE.

THEY CAN BE ARRANGED IN TWO OTHER  
REGULAR WAYS, AS IN EGG CARTONS.



DEPENDING ON HOW  
YOU LOOK AT THESE,  
YOU SEE GROUPS OF  
4 OR GROUPS OF 3.



DEPENDING ON HOW  
YOU LOOK AT THESE,  
YOU SEE GROUPS OF  
2 OR GROUPS OF 6.

WE CAN USE NUMBER SENTENCES TO DESCRIBE  
THE TOP SET OF 12.

$$12 \div 12 = 1, 1 \times 12 = 12, 12 \div 1 = 12, 12 \times 1 = 12$$

1 AND 12 ARE CALLED FACTORS OF 12  
SINCE THEY CAN BE USED IN EQUAL GROUPS  
TO MAKE THE MULTIPLE, 12.

THEY ARE NOT IMPORTANT FACTORS.

ANY NUMBER HAS FACTORS OF 1 AND ITSELF.  
(ANOTHER RULE TO REMEMBER.)

THE LOWER LEFT SET  
SHOWS US THAT 12  
ALSO HAS THE  
FACTORS 3 AND 4.

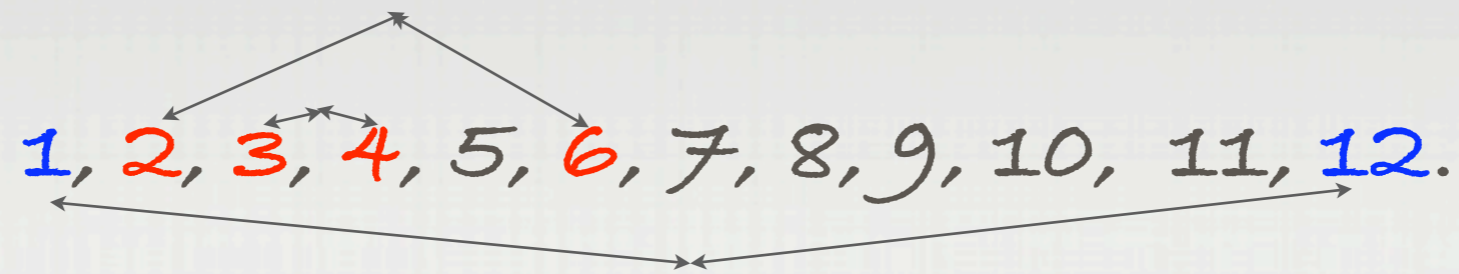
$$12 \div 3 = 4, 4 \times 3 = 12$$

$$12 \div 4 = 3, 3 \times 4 = 12$$

THE LOWER RIGHT SET  
SHOWS US THAT 12  
ALSO HAS THE  
FACTORS 2 AND 6.

$$12 \div 2 = 6, 6 \times 2 = 12$$

$$12 \div 6 = 2, 2 \times 6 = 12$$



EACH FACTOR HAS ANOTHER IT IS ALWAYS RELATED TO, JUST AS IN + AND -.

BUT NOW NOT ALL THE NUMBERS IN A WHOLE CAN PAIR UP AS THEY DO IN + AND -.

FOR 12, 5 CAN'T BE A FACTOR, NOR CAN ANY NUMBER BETWEEN 6, WHICH PAIRS WITH 2, AND 12.

THE FACTORS OF 12 ARE: 1, 2, 3, 4, 6, 12. THOSE IN RED ARE THE ONES A PERSON HAS TO REMEMBER.

THE OTHER TWO HAPPEN ALL THE TIME SO THEY CAN BE REMEMBERED BY A RULE, ONE ALWAYS WORKS WITH THE WHOLE.

FOR 12 THE IMPORTANT FACTORS ARE: 2,3,4,6.

SOME NUMBERS HAVE **NO** IMPORTANT FACTORS.

THESE ARE CALLED PRIME NUMBERS.

THEY ARE MULTIPLES OF ONLY 1 AND THEMSELVES.

SUCH NUMBERS ARE: 2, 3, 5, 7, 11, 13, .....

YOU DON'T HAVE TO LEARN FACTS FOR THESE.

ALL OTHER NUMBERS ARE CALLED COMPOSITE.

THEY CAN BE MADE INTO EQUAL GROUPS, BESIDES  
ONE AND THEMSELVES.

**COMPOSITE NUMBERS HAVE IMPORTANT FACTORS.**

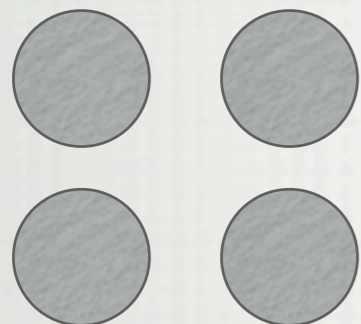
SUCH NUMBERS ARE:

4:2; 6:2,3; 8:2,4; 9:3; 10:2,5; 12:2,3,4,6; 14:2,7; ...

NOTE THAT SOME MULTIPLES HAVE  
ONE IMPORTANT FACTOR WITHOUT A PARTNER,  
SO IT MUST PAIR WITH ITSELF.

4 IS MADE WITH 2 GROUPS OF 2.

$$4 \div 2 = 2, 2 \times 2 = 4$$



THIS REGULAR ARRANGEMENT OF 4  
IS A SQUARE.

4 CAN BE CALLED THE SQUARE OF 2.

$$9: 3.$$

16: 2, 4, 8. WHERE 4 PARTNERS WITH ITSELF.

(IF WE WORKED WITH MULTIPLES AT A MORE  
ADVANCED LEVEL WE WOULD FIND THAT, LIKE 4,  
NUMBERS CAN HAVE NOTHING BUT PRIME NUMBER FACTORS.

8 CAN BE MADE OF 2 GROUPS OF 2 GROUPS OF 2.

12 CAN BE MADE OF 2 GROUPS OF 2 GROUPS OF 3.

16 CAN BE MADE OF 2 GROUPS OF 2 GROUPS OF 2 GROUPS OF 2.)

$$C - A = B, B + A = C$$

$$C - B = A, A + B = C$$

$$C \div A = B, B \times A = C$$

$$C \div B = A, A \times B = C$$

THESE 8 NUMBER SENTENCES ARE A QUICK WAY OF SHOWING ALL THE SITUATIONS THAT CAN HAPPEN WITH REAL THINGS WHERE THERE ARE TWO RELATED NUMBERS, **A** AND **B**, TO BREAK UP OR CREATE THE WHOLE, **C**.

IN ADDITION/SUBTRACTION, **A** AND **B** ARE REAL PARTS OF THE WHOLE, **C**.

IN DIVISION/MULTIPLICATION, ONLY ONE OF **A** AND **B** IS A REAL PART OF THE WHOLE, **C**.

THE OTHER COUNTS THE NUMBER OF THOSE PARTS.

I HOPE THIS HAS MADE MY IDEAS A LITTLE  
SIMPLER.

CHILDREN CAN LEARN THAT THE PARTS  
IN - AND + WORK CAN SWAP PLACES.

THEY CAN LEARN THAT THE FACTORS  
IN  $\div$  AND  $\times$  WORK CAN SWAP PLACES.

SEEING  $5 = 2 + 3$ , OR  $2 = 12 \div 6$  IS NOT A BIG PROBLEM.

IT IS ALSO NOT SCARY TO SEE A REAL LIFE  
PROBLEM WHERE THE ANSWER ASKED FOR  
IS NOT IN THE USUAL PLACE:

$$6 - ? = 4, \quad 2 + ? = 5, \quad 5 = 10 \div ?, \quad 12 = ? \times 4$$

CHILDREN LEARN THE MEANING OF THE NUMBERS  
IN EACH PLACE.

I REALIZE THERE IS A LOT MORE TO THIS  
THAN I HAVE HERE.

MY FEELING IS WE CAN STICK TO VERY  
SIMPLE EXAMPLES AT FIRST AND BRANCH OUT  
FROM THESE LATER.

DIVISION IS ONE PLACE WHERE SOME MIGHT  
WISH I HAD SAID MORE.

HERE  $12 \div 2 = 6$  CAN BE SEEN AS 12 BEING MADE  
INTO 2 GROUPS OF 6.

AFTER LEARNING FACTORS CAN CHANGE PLACES  
IN SIMPLE NUMBER SENTENCES,

IT IS NOT HARD FOR CHILDREN TO LEARN  
FACTORS CAN ALSO CHANGE JOBS IN REAL LIFE.

THE END