Simple Patterns

Young children seem to enjoy patterns, something that repeats again and again in a dependable way.

This may reflect a basic desire for order in their life, without taking away from their enjoyment of novelty.

Patterns can make learning arithmetic skills easier.

They can make the learning of facts more enjoyable.

Playing with them can lead to a lot of enjoyable practice.

They can let a child know when an error has been made simply because a pattern is broken.

The patterns that follow are simple; they are there because of the way our number system works.

Some arise when we do something quick and simple to numbers.

1. Counting. Patterns arise in writing and naming numbers.
2. Odd and Even numbers in addition, subtraction, multiplication, and division.
3. Patterns that arise when we subtract out all the nines in numbers to see what remains.

I have shown more than you might want to use for the patterns.

Children might like to be introduced to some patterns, and then discover the rest themselves.

You'll have to decide.

1. **Counting** may be a young child's first activity involving numbers. This does involve novelty, and repetition.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are the only shapes we need to record a number of things.

[ ], [x], [xx], [xxx], ...

These shapes, numerals, are used over, and over, in special ways, to represent larger and larger amounts.

0 is a unique amount. It represents nothing to be counted!

00, 01,02, ... are exactly the same as 0, 1, 2, ...

It is when we put a 1 before the other numerals that we start a Place Value system.

10, 11, 12, ...

Before the single 0, 1, 2, 3, ... only counted single things, [ ], [x], [xx], [xxx], ...

In this case, 10, the 1 on the left represents the amount ten [xxxxxxxxxx], plus what is to the right.

10 is a way of saying there is a group of [xxxxxxxxxx].The 0 on the right tells us there is a place for a numeral. Nothing is in it,

BUT because the 0 is there we can see the 1 is in a new place, the ten's place.

12 is a way of naming the amount of x's here: [xxxxxxxxxx] + [xx].

The place the 1 sits in is called the ten's place and where the 2 sits is the one's place. The one's place may have nothing in it,

but it must be shown to be there! Otherwise 10 would look like 1. The 0 holds the place.

Using this system, 1 has more value where it sits in 12 than 2 does.

Children find this amusing, and puzzling, at first.

02 may just be 2, but 20 counts this much: [xxxxxxxxxx] [xxxxxxxxxx].

At first 29 is, and may remain for some time, a mystery to some.

How can 2, where it sits, be counting more than 9?

This lack of understanding of the power of place value can continue to cause problems in later work,

especially in addition and subtraction of two place and larger numbers.

A good grasp of the workings of the place value system is important throughout life.

Even adults may be unaware that a billion is a thousand times bigger than a million.
This can lead to severely underestimating the cost of government's proposed expenditures, for example.

This shows a failure to grasp the pattern basic to the place value system.

100 is larger than 99 because 9 sets of 10, plus 9 more, are one less than 10 full sets of 10.
1 in 100 sits in the third place to the left, the hundreds place.
As the places move to the left, their value increases by ten times.
1 is [x],
10 is [xxxxxxxxxx],
100 is [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx] [xxxxxxxxxx].

Now another fact about our place value number system.
The ones, tens, and hundreds places are repeated over and over again, but their value depends on the place occupied by this set of three.
The value of this set of three places is shown by the number of places to their right.
In 1,000, the 1 is read as one again, but since there are three places to its right, we call it one thousand.
In 10,000 the first two figures are read as ten, plus the word thousand, or ten thousand.
In 100,000 the first three figures are read as one hundred, plus the word thousand, or one hundred thousand.
I've used a comma to separate the sets of three places, but this is not always done.

(Children have to learn that some people read 1,400 as one thousand, four hundred,
But some may write 1400, and read it as fourteen hundred.)

Another peculiarity of our number system is that after 10, we, until 20, give single names to numbers.
11 is eleven, 12 is twelve, 13, is thirteen, ... These names are exceptions to the general system, yet they must be mastered.

Those who say fourteen hundred are treating fourteen as a single amount sitting in the hundred's place.
14,000 is read as fourteen thousand, but 140,000 is read as one hundred and forty thousand.
It is when we get to twenty the common pattern of the system returns.
20 is twenty, 200 is two hundred, 2000 is 2 thousand, (rarely 20 hundred), 20,000 is twenty thousand.

21 is twenty-one, 22 is twenty-two, ... 51 is fifty-one, 52 is fifty-two, ...
This way of naming the numerals follows the same pattern from 20 on.
The number in the ten's place is given a name related to its name in the one's place.
A hyphen is used to add any single digit numeral's name, other than 0.

121 is read as one hundred and twenty-one, although some leave out the word "and".
Apart from the peculiar 11 to 19 (219 is read as two hundred and nineteen.),
this is how this set of three place are read, no matter how many sets of three places are to their right.

121,000 is read as one hundred and twenty-one thousand.

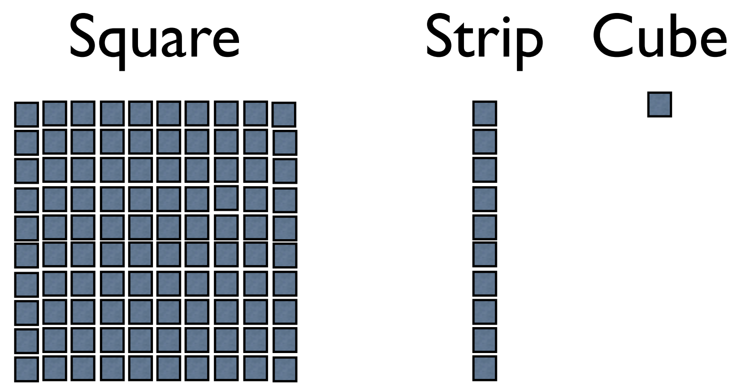
Once children can master reading this set of three places, they can enjoy the feeling of power that comes from reading really big numbers.
This can be done with a few new words, thousand, million, billion, ...

121,000,000 becomes one hundred and twenty-one million with two sets of three places to the right of 121.
121,000,000,000 becomes one hundred and twenty-one billion with three sets of three places to the right of 121.

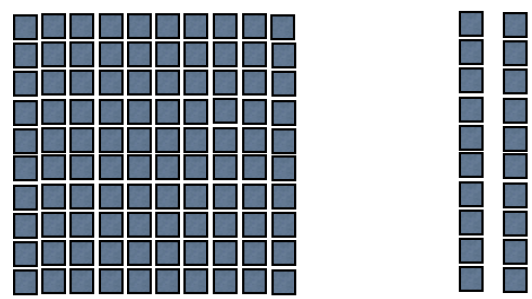
This is not to say that children understand the value of a thousand, million, or billion. That takes experience.
I was lucky to have more than 1000 one inch wooden cubes available for use in my classroom.
We could put down one cube, and beside it to the left we could have a strip of ten cubes,
and again to the left a set of ten strips of 10 cubes, which formed a square of 100 cubes

This way we could visualize the set of three places that are used over and over again in reading large numbers.
A little to the left again we built ten layers of hundred squares, one thousand cubes.
Since this set formed a new cube,10 by 10 by 10, and was named one thousand, we could begin to see the pattern.
A set of ten one thousand cubes would form a strip of ten, 10,000.
Ten strips of one thousand cubes would form a square, 100,000.
Ten squares of one thousand cubes piled together would make a new cube, 1,000,000 ....
So the pattern of blocks, moving from the right to the left was:
.... cube, square, strip, cube, square, strip, cube, square, strip, cube, square, strip, cube!

Visualizing the beginning of this pattern gave children a start at seeing how quickly the value of numerals grew
when placed farther and farther to the left.



Hopefully you can now see how the reading of the numerals in sets of three places makes some sense.
The first set of three places are as you see above.
(121 would be one square of one hundred cubes (Sorry, I can only show squares), two strips of ten cubes, and a single cube.)



Thinking about ten squares of ten strips of 10, at the top of this page, forming a new cube, a thousand times larger than the first cube, gives some some idea of the growth in value of the places for numerals as they move farther left.
To the left of the new cube, 1,000, would be a strip of ten such cubes, 10,000, and then a square of one hundred such cubes,100,000.
Next to the left would be a cube of ten squares of one thousand cubes, 1,000,000, or one million.
Think of three more places to the left, a new cube a thousand times bigger, 1,000,000,000, or one billion.
If these blocks were 1 inch cubes, the first first new cube would be 10 inches on each side, the next cube 100 inches, then 1000 inches.
If each first cube weighed 1 ounce, the first new cube would be 1000 ounces, and so on.

Before leaving counting patterns you should be aware that the pattern continues to the right of the one's place for older children, when they start the decimal system, and usually that involves money.
It can be a bit confusing at first since to the right of the one's place, shown by a period or decimal point, the pattern is a mirror image of what is to the left.

Hundred Tens ones. Tenths hundredths

317.56

Still, starting at the hundredths place and going left, each place remains 10 times the value of the place to the right.

2. **Odd and Even** are simple terms children can easily learn. They are related to sharing. Two children can easily share four cookies. Give them five cookies and there is a problem. Give four cookies to three children and there is more of a problem sharing them equally. And children have a keen sense of what is fair in sharing.
We call two, four, six, ... The even numbers. One more, or one less, are the odd numbers.

Even numbers of children allow each child to have one partner, odd numbers don't.

There is a basic pattern to odd and even numbers.
We could say 0 is even since shared with a partner it allows each to have the same amount.
1 is odd since it has to be broken up to share with a partner.
2 is even since each partner can have 1.
Even, odd, Even, odd, Even, odd, Even, odd, Even, odd, Even, odd, ...
This is a simple pattern to see, useful in learning to count, but it isn't the most useful one.

What happens when two even numbers are added? The answer is always even. It will be, or will end with an even number.
8 + 8 = 16, and 6 is even. Get an odd number answer and you have an error. This is even true for 46 + 38 = 83

What happens when two odd numbers are added? Think of odd numbers as being even plus one.
Put two odd numbers together and the two "plus ones" make the answer even. 3 + 3 = 6
If you were to add three odd numbers together this would not work as there would be an extra 1 to add on.
Can you think of a rule for adding odd numbers?7

Odd and even can be used to check subtraction, the opposite action to addition.
When an even number is taken from an even number, the result will be even.
When an odd number is taken from an even number, the result will be odd.
Take an even number from an odd number and the result is odd.
Take an odd number from an odd number and the result is even.
Try them.

If you think of even numbers and multiplying, or dividing, you should remember these are fast ways of adding or subtracting equal groups.

In 5 x 6, we are either putting together groups of an even number, or an even number of odd numbered groups.
The answer will be even.
In 4 x 8, we are putting together groups of an even number, either way we look at it.
The answer will be even.
In 5 x 7, we are putting together an odd number of odd numbered groups, either way we look at it.
The answer will be odd.

If we have 16 ÷ 8, we have an even number being taken out of an even number.
The answer could be even or odd. It is even.
If we have 20 ÷ 5, we have an odd number being taken out of an even number.
The only way this can happen is an even number of times.
In 28 ÷ 4 we have we have an even number being taken out of an even number.
The answer could be even or odd. It is odd.
In 35 ÷ 5 we have an odd number being taken out of an odd number.
The answer will be odd.

Obviously odd and even is more helpful with +, -, or x.

3. Casting out Nines. This simple activity may seem strange at first. It works because 9 = 10 - 1, but an explanation is not needed.
<u>You just have to accept that it works</u>.
Because <u>we remove any 9's</u>, **all** numbers will group themselves according to what is left over: 0, 1, 2, 3, 4, 5, 6, 7, 8.
9, 18, 27, 36, 45, ... Are all part of the group that has 0 left when all 9's are taken out.

When you use this you must forget about the value a number has because of its place.
The 9 in 90 may be worth 9 tens, but in this game like activity it is worth 0. Even in 99, the nines are just 0.
In 90 we can take out ten 9's, and in 99 we can take out eleven 9's, but 0 always remains.

<u>That is the way this game works</u>. Place value makes no difference.
In 439, the 4 is just 4 (the amount remaining if we take out all 9's), the 3 is just 3, and to 9 is 0.
But the activity called for is to add the numerals you see, 4, 3, and 9 (which is 0).
The total is 7.
In this game, 439 is part of the group with 7 remaining when all 9's are removed.
<u>Any number is like 0, 1, 2, 3, 4, 5, 6, 7, 8, but not 9.</u>

Before going on, I'll show you one use.
Say we have to add 439 and 218.
439 is like 7 since the 9 counts as 0. 218 is like 2, since 1 and 8 make 9, and 9's are 0.
Now instead of adding 439 and 218, we take the 7 and 2 and add them. This makes 9, which is 0.

This may seem a strange activity, but if we do the real adding of 439 and 218, we will get 657.

Now cast out nines in 657. 6 and 5 make 11, 1 and 1 make 2, 2 and 7 make 9, or 0.
Remember 439 was like 7, 218 was like 2.
If we add what each is like, and throw out nines, we get 0, just like the answer to 439 + 218.

If we had made a small adding mistake, perhaps an answer of 659, we would find the total, after casting out nines in 659, would be 2.

<u>Only if casting out 9's in the two or more parts gets you to the same number as casting out 9's in the whole can the answer be correct.</u>

<u>It can also be wrong</u>, but you would have to make a mistake equal to some number of 9's.

This will work for any number of amounts added together, and in fact you can choose the order you want to add the casting out 9's amounts.
In 356 + 418 + 567 + 208 =?
you can quickly get rid of anything that makes 9 if you keep careful track of what you skip over.
From the left, 3 and 6 are 9, the 5 you skipped over and 4 are 9, 1 and 8 are 9, 5 and 6 is 11, the same as 2, 2 and 7 are 9, 2 and 8 are 10, the same as 1.
Whatever the answer, casting out nines should get 1. If it doesn't the answer is wrong. If it does it is probably correct.

<u>This is a quick way of picking up wrong answers</u>. It will not <u>guarantee</u> correct answers, only wrong answers.
Try the question above yourself.

Casting out nines is easiest to use for addition, and it is best used for adding large numbers where little mistakes can be made.

It can also be used for subtraction, if you remember subtraction is the opposite action to addition.

87 - 15 = 72. Doing the opposite would be putting together 72 and 15, which should get us back to 87.
Applying the casting out nines method for addition, we look at 72 + 15 = 87.
7 and 2 are like 0, 1 and 5 are like 6. 0 and 6 are like 6 so 87 should also reduce to 6.
8 and 7 are 15, 1 and 5 are 6. The two casting out nines amounts are the same.
87 - 15 = 72 is probably correct.

2943 - 317 = 2516. Right or wrong?

Combine the casting out 9 values for 317 and 2516. They should be the same as that for 2943.
317 + 2516: 3 and 1 is 4, 7 and 2 is 0, 4 and 5 is 0, 1 and 6 is 7. Is this the same as 2943?
2943: skip the 9, 2 and 4 and 3 make 0. **My answer is wrong**.
Try 2943 - 317 yourself.

Working backwards to check an answer is more difficult for young children.

No matter what, children will get a lot of practice trying this out.

Casting out nines can also be used for multiplication, and division.
Multiplication is a quick way of adding equal amounts.

Just follow these examples:

284 x 415 = ?
Well, 284 in casting out nines is 5, 415 is 1.
Multiply 5 times 1 to get 5.
284 x 415 = a number that will be 1 after casting out nines.
284 x 415 = 117860
117860: 1,1, and 7 are 9, 8 and 6 and 14, 1 and 4 are 5.
The answer is probably correct.

Division is a quick way of subtracting equal groups.

For division it is easiest to work with no remainder.
Lets make it simple and check 117860 ÷ 284 = 415.
Here we have to remember division is opposite to multiplication, so we do the reverse.
We would check to see if 415 x 284 did make 117860.
We've really done this already, so that's quick.

So far, casting out nines has just been a quick way of checking for simple errors.

One more activity provides practice, and some interesting patterns related to what we just did.

The pattern that is most interesting occurs most obviously in the multiplication “tables”.

Take the nine times table answers in order: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, ...

Note that if we treat all these the casting out nines way, we first get 9 for each number, which is the same as 0.

This means that 756 x 9 has to equal an amount that reduces to 0 by casting out 9’s.
If it does it may be correct, and if it doesn’t it certainly is wrong.
(If you look at the figures in the one’s place, left to right, and it the tens place, right to left, you’ll see another pattern.)

The eight times table: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...
reduces to 8, 7, 6, 5, 4, 3, 2, 1, 0, 8, 7, 6, ...
Interesting, but not too useful.

The seven times table: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, ...
reduces to 7, 5, 3, 1, 8, 6, 4, 2, 0, 7, ...
Interesting, but not too useful.

The six times table: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...
reduces to 6, 3, 0, 6, 3, 0, 6, 3, 0, 6, 3, 0, 6, ...
Obviously the answer to anything multiplied by 6, when casting out nines is used, has to reduce to a 6, 3, or 0.

The five times table: 5, 10, 15, 20, 25, 30, ...
shows that any to a number multiplied by 5 has to end in 0 or 5. This is useful.
But using casting out nines we see a pattern of 5, 1, 6, 2, 7, 3, 8, 4, 0, 5, 1, ...
Interesting, but not too useful.

The four times table: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...
reduces to 4, 8, 3, 7, 2, 6, 1, 5, 0, 4, ...
Interesting, but not too useful.

The three times table: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...
reduces to 3, 6, 0, 3, 6, 0, 3, 6, 0, 3, ...
This produces the same useful result as the six times table.

The two times table : 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...
reduces to 2, 4, 6, 8, 1, 3, 5, 7, 0, 2, ...
Interesting, but not too useful.
The useful point here is that a number multiplied by two has to equal an even number.

To repeat, patterns can be useful, or just interesting.

Casting out nines is an activity that can be useful, or can bring out an interesting pattern.

Whatever the reason for using these ideas, they can provide a reason for some interesting practise.