CHAPTER 4: GRAVITY EXTENDS THROUGHOUT THE UNIVERSE

4.1 – Gravitational Forces due to Earth

EXERCISE 4.1

1. List the four fundamental forces of nature. Which is the weakest?

2. How is gravitational force an example of Newton’s third law?

3. State the equation (with units) for gravitational force or weight.

4. How is gravitational mass determined? Is this the same as inertial mass? Indicate how it is the same, and how it is different.

5. A rock has a mass of 5.40 kg, and a weight on earth of about 53.0 N. What is the rock’s weight on the moon, where g equals about 1.61 N/kg? What is the rock’s gravitational mass on the moon?

6. In deep space, or in orbit around the earth (in the space shuttle, for example), g seems to be effectively zero. What would the mass of the rock in question #5 be in this situation? If you were an astronaut in the shuttle, how would you determine this answer?

7. How could you change the mass of the rock in question #6? How could the weight of the (original) rock change?
8. Define the term **field** as used by physicists. What does the concept of a field help to explain?

9. Apply the definition of field to the **temperatures** at different points around a candle flame. Does this field seem to have a direction?

10. Apply the definition of field to the magnetic attraction shown by a magnet for iron filings placed around it. (The picture to the right shows such a field. To make this picture, two magnets were placed underneath a sheet of paper, and iron filings were sprinkled on top of the paper.) Does this field seem to have a direction?

11. Which of the fields in question #9 or #10 is a **vector field**? Which is a **scalar field**? Give one more example of each kind of field.

12. Sketch, side-by-side, the gravitational field surrounding the earth and the one surrounding the moon. How do your sketches show that the earth’s field (at the earth’s surface) is about six times as strong as the moon’s field? How do your sketches show that the strength of both fields **decreases** as you move away from the earth or the moon?
13. How is the direction of a gravitational field defined?

14. Define **gravitational field strength**: give the equation, with units. (Note that this is an *empirical* or *experimental* form of a more general relationship. The next question illustrates how an experiment can be performed to measure \( g \).)

15. The sketch shows a mass suspended from a spring balance near the surface of distant planet Trof. What is the gravitational field field strength on the surface of Trof?

16. What would a mass of 320 kg weigh on the surface of Trof?

17. As one moves further from the centre of the earth, the gravitational field strength decreases. Describe this decrease **mathematically**. Sketch the shape of the graph that shows this decrease.

18. Use the sketch to find the gravitational field strength at points X, Y, and Z. Remember that the gravitational field equation shows an *inverse square relationship* to distance. For example, doubling the distance reduces the field strength to \( \left( \frac{1}{2} \right)^2 = \frac{1}{4} \) of what it was before.
4.2 – Newton’s Law of Universal Gravitation

The gravitational field strength relationship is just a special case of Newton’s law of universal gravitation, which says that any two mass objects in the universe have a mutual force of attraction acting between them. Newton discovered that this force depends directly on the masses involved (larger masses => larger forces), and varies inversely with the distance between the centres of the masses (making it similar to gravitational field strength.)

EXERCISE 4.2

1. Sketch a graph showing how gravitational force varies with mass; choose mass as the independent variable. Note that you are assuming the existence of a second, constant mass with this graph, and that separation between the masses doesn’t change.

2. Sketch a graph showing how gravitational force varies with distance between the centres of the masses. Note that you are assuming that both masses remain the same for this graph.

3. Write Newton’s universal law of gravitation as an equation. State units; include a value for the universal gravitation constant, \( G \).

4. Examine figure 4.13 on page 204. What gravitational force (in terms of \( F_g \)) would exist in each of the following situations?

   a) \( m \) and \( 4m \) at distance \( r \) ____________

   b) \( m \) and \( 4m \) at distance \( 2r \) ____________

   c) \( 2m \) and \( 3m \) at distance \( r \) ____________

   d) \( m \) and \( m \) at distance \( 5r \) ____________

   e) \( m \) and \( m \) at distance \( \frac{1}{3}r \) ____________

   f) \( 3 \, m \) and \( 4 \, m \) at distance \( 3r \) ____________
5. Who first determined the value of $G$ for objects on the surface of the earth? Describe in general terms how he was able to measure the very small forces involved.

6. A device like the one described above is used to measure $G$; a force of gravitational attraction of 7.82 nN is obtained. If the masses used were 15.0 kg and 2.00 kg, and they were placed 50.0 cm apart, what value of $G$ was determined? Determine a percent error in this answer, compared to the accepted value. Use the formula below to find percent error.

$$\% \text{ error} = \left( \frac{\text{measured value} - \text{accepted value}}{\text{accepted value}} \right) \times 100$$

Because every object exerts a gravitational force on every other object, over an infinite distance, it is not really correct to say, “There is no gravity in space.” There are points, however, where the net force of gravity is effectively zero. For example, between the earth and the moon are points called Lagrange points, where the attractive force from the earth on an object is equal but opposite to that from the moon. (There are other Lagrange points between the earth and the sun, etc.)

7. Lagrange point L1 for the earth and the moon is about 323 000 km from earth. What forces would act on a spaceship with a mass of 40 000 kg at this point, caused by the earth? Caused by the moon? (See chart on page 209 for values.) Draw a free-body diagram of the spaceship at Lagrange point L1.

8. Find the net gravitational force on the spaceship in question #7 if it is halfway from the earth to the moon. (See example 4.3 on page 209.)
9. Somewhere in deep space, three masses are arranged as shown. Find the net gravitational force (including a direction, left or right) on mass \( m_1 \) caused by the other two masses.

\[ m_2 = 10000 \text{ kg} \quad 10.5 \text{ m} \quad m_1 = 2000 \text{ kg} \quad 7.50 \text{ m} \quad m_3 = 20000 \text{ kg} \]

10. Find the net gravitational force on mass \( m_2 \) (in question #9) if it exchanges positions with mass \( m_1 \).

11. Use the techniques in example 4.2 on page 208 to determine the force of gravity in each situation. The sketch shows the initial conditions.

a) Object A’s mass increases 3 times and the distance changes to 2.00 m.

b) Both object A and object B have their masses halved; the distance doubles.

c) Object B’s masses decreases to one-quarter; the distance changes to 1.00 m.

d) The distance increases to 100 m.

e) Both masses increase ten-fold.
12. Why do the water bodies of the earth experience tides?

13. Explain the principle of **gravity assist** in interplanetary travel.

### 4.3 – Relating Gravitational Field Strength to Gravitational Force

The weight of an object on the surface of the earth can be found using \( F_g = mg \), where \( g \) is the earth’s **gravitational field strength** – the force exerted on the object per kilogram of mass. The value of \( g \) is usually obtained by measurement, but it can also be calculated using Newton’s law of universal gravitation.

Start with the universal law of gravitation; \( m_1 \) will be a one-kilogram mass on the earth’s surface, and \( m_2 \) will be the mass of the earth. The value of \( r \) will be the distance between the centre of the one kilogram mass and the centre of the earth – essentially the earth’s radius.

\[
\frac{F_g}{m_1} = \frac{Gm_1m_2}{r^2}
\]

Divide both sides by \( m_1 \). The left-hand side of the equation now expresses the force on one kilogram of mass: N/kg.

But N/kg is the unit for gravitational field strength; this expression is actually calculating \( g \), so replace \( \frac{F_g}{m_1} \) with \( g \). Because there is only one mass term remaining in the equation (\( m_2 \) - the earth’s mass), we can drop the subscript. What remains is a formula for finding the gravitational field strength caused by mass \( m \), at a distance \( r \) from the centre of mass \( m \).

**EXAMPLE – CALCULATING \( g \)**

The asteroid Vesta in the main asteroid belt of our solar system has a mass of about \( 3.06 \times 10^{20} \) kg and a radius of \( 2.70 \times 10^5 \) m. Assume that Vesta is spherical in shape. (It’s actually an oblong, football shape.)

a) Find the value of \( g \) on the surface of Vesta.

\[
g = \frac{Gm}{r^2} = \frac{6.67 \times 10^{-11} \times 3.06 \times 10^{20}}{(2.70 \times 10^5)^2} = 0.28008 \text{ N/kg}
\]

b) An astronaut in full gear on earth has a weight of 2300 N. What would this astronaut weigh on the surface of Vesta?

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b) An astronaut in full gear on earth has a weight of 2300 N. What would this astronaut weigh on the surface of Vesta?
You could first find the mass of the astronaut using \( F_{\text{g-earth}} = mg_{\text{earth}} \); using the equation again with \( g \) for Vesta would give the weight on Vesta. Or, you could use ratios:

\[
\frac{g_{\text{earth}}}{g_{\text{Vesta}}} = \frac{F_{g-\text{earth}}}{F_{g-\text{Vesta}}} \quad \frac{9.81}{0.28008} = \frac{2300}{F_{g-\text{Vesta}}}
\]

\[F_{g-\text{Vesta}} = 65.666 \text{ N} \]

\[F_{g-\text{Vesta}} = 65.6 \text{ N} \]

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EXERCISE 4.3

1. Describe how the value of \( g \) on earth changes as you move north and south of the equator (which is 0° latitude.) Why does this occur?

2. The rotation of the earth also changes the measured value of \( g \). If the earth were perfectly spherical, would you apparently weigh more or less at the equator than at the poles? Explain.

3. The earth’s crust is thicker under large mountain ranges, because it has to support the mass of rock that makes up the mountains. How would you expect \( g \) to change if you were near the Rocky Mountains than if you were on the grasslands of southern Alberta? Why would you expect this to happen?

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On pages 222-226 is a discussion of true weight versus apparent weight. True weight is found using mass and local gravity: \( F_g = mg \). Apparent weight is the normal downward force you exert on a scale (or the upward force the scale exerts on you.) If you and the scale are stationary on the earth’s surface, your true weight and apparent weight are the same.

4. How does your real weight compare to your apparent weight if you are moving upwards at a constant speed? If you are moving downwards at a constant speed?
5. If you’ve never ridden in a fast elevator in a tall building (say, 40 stories), imagine the sensations of weight change you would experience during the ride. How would your weight seem to change as the elevator accelerated upwards from the ground floor? How would your weight seem to change as the elevator traveled upwards towards the fortieth floor? How would your weight seem to change as the elevator slowed down as it approached the fortieth floor?

6. Repeat question #5 for the ride down from the fortieth floor to the ground floor.

When you stand on a surface – or a scale – your weight (caused by gravity) is opposed by an upwards force from the surface or scale, called the normal force. If you are stationary (or moving with a constant upwards or downwards velocity), weight and the normal force are equal but opposite. However, if you are accelerating upwards or downwards, the normal force (but not gravity) changes. Why this happens is easy to understand for upwards acceleration; the normal force has to both support you against gravity and provide an additional net force upwards to cause your acceleration.

The normal force for an accelerating frame of reference – like an elevator accelerating upwards or downwards – is given by $F_N = m(a - g)$, where $a$ is the acceleration of the reference frame and $g$ is local gravity. Remember that $g$ as always is down, and therefore would have a negative sign when substituted into this equation.

**EXAMPLE – APPARENT WEIGHT IN AN ACCELERATING FRAME OF REFERENCE**

A girl with a mass of 55.0 kg rides an elevator from the ground floor to the fortieth floor of a tall office building. The elevator initially accelerates upwards at 1.40 m/s$^2$, then travels at a constant upwards speed. As it nears the fortieth floor, the elevator slows at -1.70 m/s$^2$. Find the girl’s apparent weight (the normal force) as the elevator accelerates upwards, as it travels at a constant speed, and as it slows reaching the fortieth floor.

- **Accelerating upwards:**
  - $F_N = m(a - g)$
  - $F_N = (55.0)[+1.40 - (-9.81)]$
  - $F_N = +616.55 \text{ N}$
  - $F_N = +617 \text{ N}$

- **Constant speed:**
  - $F_N = m(a - g)$
  - $F_N = (55.0)[0 - (-9.81)]$
  - $F_N = +539.55 \text{ N}$
  - $F_N = +534 \text{ N}$

- **Slowing at fortieth floor:**
  - $F_N = m(a - g)$
  - $F_N = (55.0)[-1.70 - (-9.81)]$
  - $F_N = +446.05 \text{ N}$
  - $F_N = +446 \text{ N}$

7. A man with a mass of 85.0 kg rides an elevator from the eightieth floor of a tall building to the ground floor. The elevator initially accelerates downwards at 1.85 m/s$^2$ and eventually slows at 1.60 m/s$^2$ as it nears the ground floor. Find the man’s apparent weight as the elevator starts down and as it reaches the ground floor.
8. An astronaut with a true weight of 770 N lifts off the ground in the space shuttle. The shuttle’s initial acceleration is $3.55 \text{ m/s}^2$ upwards. Find the astronaut’s apparent weight.

9. A student riding down in an elevator slows at $1.06 \text{ m/s}^2$ as the elevator reaches a lower floor. With this acceleration, the student’s apparent weight is 540 N. Find the student’s true weight and her mass.

10. A future astronaut is about to land on the planet Mercury. Her spaceship is approaching the surface of the planet, and slowing at $3.50 \text{ m/s}^2$; the astronaut has a mass of 72.5 kg.
   a) Find the value of $g$ on Mercury. (Use the data from page 218.)

   b) Find the astronaut’s apparent weight (normal force) during the descent.

   c) What will the astronaut weigh on the surface of Mercury?

11. Describe the conditions necessary for free-fall.

12. Determine the apparent weight (normal force) of a 85.0 kg skydiver immediately after he jumps from the airplane, using the equation for normal force.

13. The environment of the space shuttle is often referred to as weightlessness. If the term actually means the absence of any significant gravitational force, why is it incorrect to say objects and astronauts in the shuttle are weightless? What is a better term to use in describing the effects seen in the shuttle or on the international space station?