Logical Verification of the Goldbach Conjecture

Abstract: the Goldbach conjecture, from 1742 is the longest standing problem in the history of mathematics. I am here proposing that the validity of the Goldbach conjecture is axiomatic to the number system itself. I am also proposing that stronger axiomatic (Goldbach-related) conjectures are applicable to all numbers (even, odd and prime) and that these equidistance conjectures lie at the heart of the Goldbach Conjecture[1] and its verification. Included in this article (version #2) is a dissertation on the Goldbach comet and its upper and lower bounds.

Following my argument for axiomatic consistency (of the number system itself) my proof contains two essential components or "planks:"

#1. Comprehensive Targeting (of primes for Goldbach pairing)
and;
#2. Comprehensive Availability (of primes for Goldbach pairing)

Key words: Goldbach conjecture, primes, prime field, effective conditioning primes, largest effective conditioning primes, prime curve, Goldbach Comet, High Yield Evens, Low Yield Evens, number theory.

"Comprehensive**" …**here means that the conclusion (targeting or availability) applies to the number system as a whole.

Equidistant spacing is axiomatic for the whole number system (two is necessarily equidistant from both one and three). All prime multiples are equidistant (up-down symmetric) 3,6,9,12. Six is equidistant from both three and nine. The adjacent remainders of these, up-down symmetric multiples (of say, "3") are also up-down symmetric (4-5, 7-8, 10-11 etc.) The "primes" are necessarily, the up-down symmetric remainders (the perfect negative image) of the set of all up-down symmetric smaller prime multiples -just as the set of all odds is the "perfect negative image" of the set of all evens. I maintain that the validity of the Goldbach conjecture is critically dependent upon the conservation of up-down symmetry or "equidistance." (plus prime availability produced by system finiteness).

This analysis treats every ***even number*** *as the finite end point of a subordinate finite number system.*

The Goldbach Conjecture:

"Every even integer greater than 2 can be expressed as the sum of two primes"

Instead of focusing on the even number itself, my analysis starts by addressing the mid-point of all even numbers. The mid-point for the number 20 for instance is 10 and the Goldbach pairs (for the number 20) are necessarily equidistant from this mid-point. I.E., 10+**3** =13 and 10–**3** =7. This is what I mean by "equidistance" or "up/down symmetry." It is upon this internal symmetry that the validity of the Goldbach conjecture is based.

The "T-Diagram"

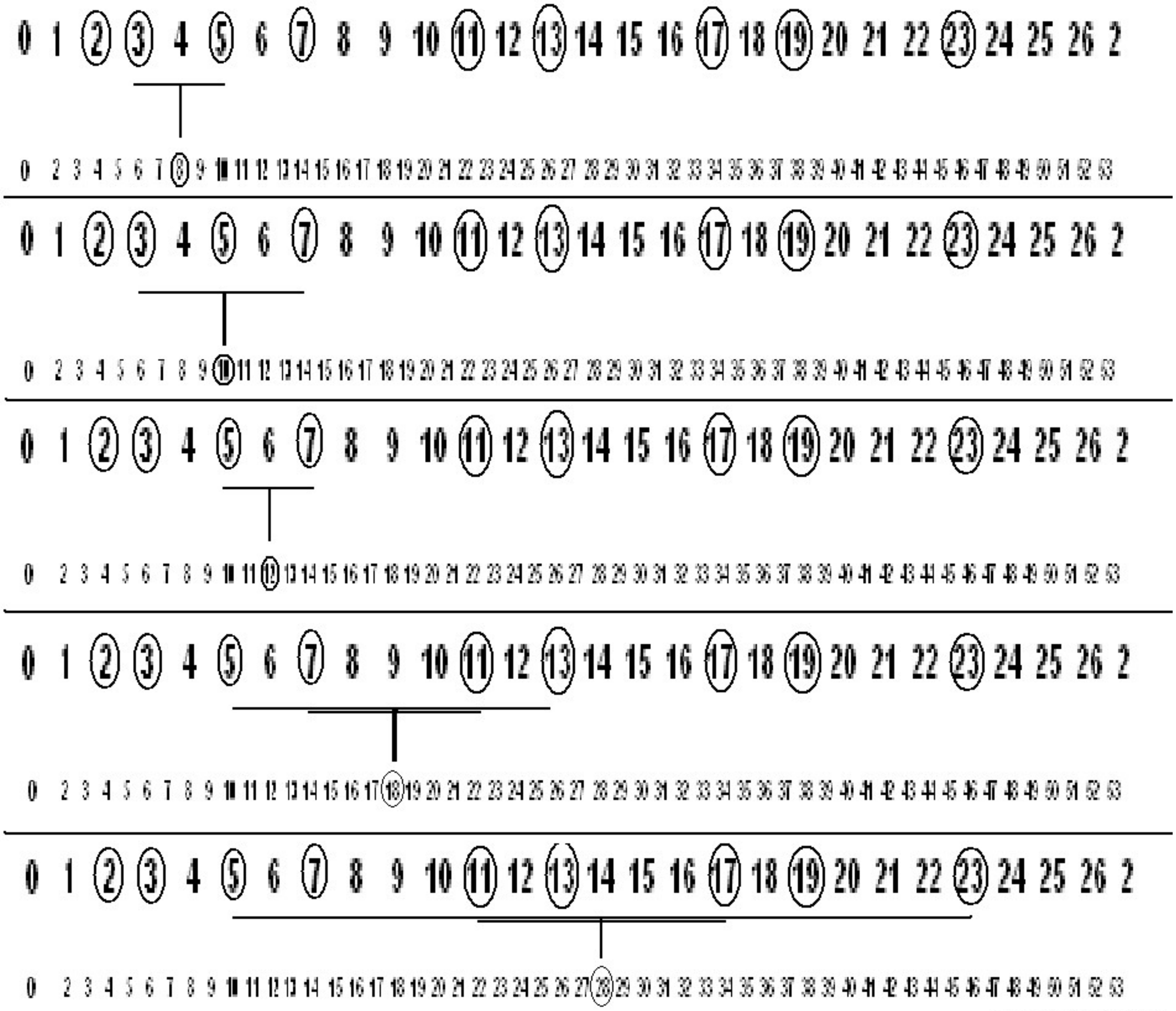


Fig #1. *The "T-diagram" demonstrates the concept of "midpoint equidistance." The Goldbach pairs for 8, 10, 12, 18 and 28 (all even #'s) are here displayed. The* <u>*midpoints*</u> *for these evens respectively here are 4(even)5(prime)6(even)9(odd) and14(even).*

While it is possible to consider the number system *as a whole* (with the even numbers being just components of the single large system) it is also possible to consider each even number as an <u>end point</u> of a smaller system with its attendant resolution problem and system finiteness.

With system-finiteness (restricting divisibility and subsequently) producing primes within each of these smaller systems --- and the equidistance axioms of the number system itself forcing the primes into Goldbach pairs, the Goldbach conjecture can be verified, not by secondary mathematical proof, but by the fundamental logical axioms of the number system itself.

Equidistance Conjectures

I am hereby proposing that a stronger conjecture (hereafter "The Equidistance Conjecture #1") is applicable to all numbers (even, odd and prime) greater than three.

Equidistant Conjecture #1:

"All numbers (even, odd and prime) greater than three, are equidistant from at least two discrete primes"

Notice that unlike the Goldbach conjecture that applies only to evens the equidistant conjectures, being axiomatic, apply to **all** whole numbers (even, odd or prime). Moreover if this "equidistant conjecture" is applied not to primes but to the "Prime Curve" instead, that includes "1" then the equidistance conjecture applies to all positive integers (odd, even or prime), larger than one. This is because two is equidistant from 1 and 3 and three is equidistant from both 1 and 5 – all being prime curve numbers.

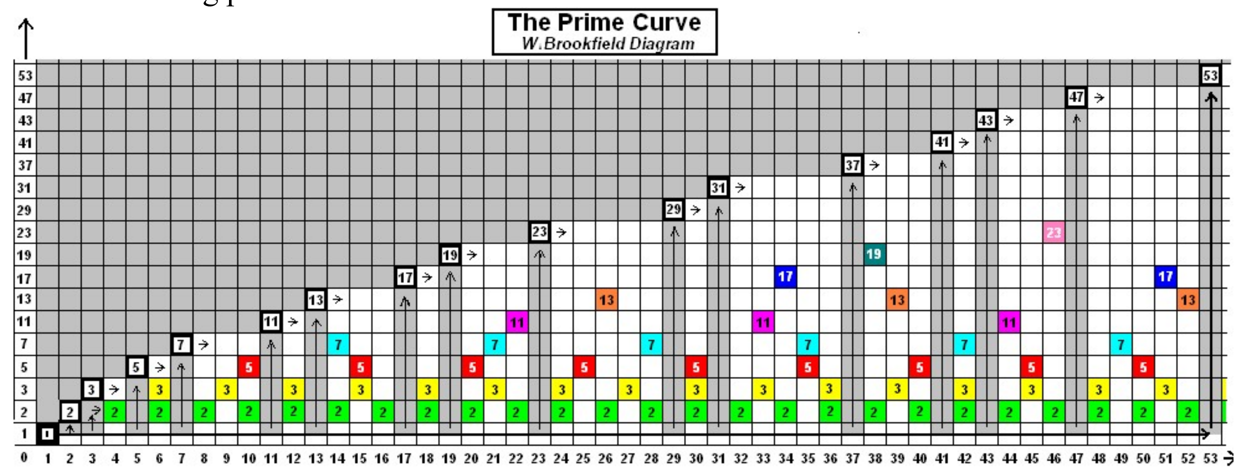


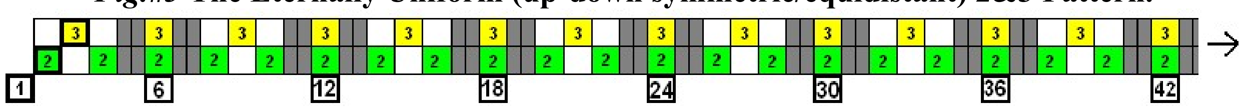
Fig #2. *The "prime curve" is produced by plotting the whole numbers (1234..) on the x axis and the primes (#'s 1234..)^2 on the y axis (or vice versa). The prime curve begins at a 45 degree angle (with 0123..) and starts to bend (curve) at 2^2 with (3,5,7) and then again with 3^2 (7,11,13.17,19,23) and then again at 5^2 with (29,31,37,41,43, 47, 53 etc,).*

This gives us a "**Generalized Equidistant Conjecture**:

"*All numbers (even, odd and prime) greater than one, are equidistant from at least two discrete prime-curve numbers.* "

I am therefore proposing that this stronger conjecture (the "Comprehensive Equidistance Conjecture") is applicable to all numbers (even, odd and prime) greater than "1" and that equidistance lies at the very heart of the Goldbach Conjecture.

Fig.#3 **The Eternally Uniform (up-down symmetric/equidistant) 2&3 Pattern.**



The completion of each 3-epicycles occurs at 6,12,24 etc.

We know that both the even and odd numbers exhibit global up-down symmetry and subsequent global "equidistance consistency." We however, are not as certain about the prime numbers. One reason for this uncertainty is that the occurrence of primes as one counts upward seems erratic and inexplicably "gappy." Clearly however a string of prime multiples (all multiples of say, 3, 5 or 7) is just as "equidistant consistent" as a string of whole numbers (all multiples of 1). All subsequent primes must necessarily appear in the equidistant gaps between former equidistant prime multiples. Primes are equidistant and *out-of-phase* whereas prime multiples are equidistant and *in-phase*. The evolution of numerous prime cycles is necessarily a complicated epicycle but an epicycle is a cycle nonetheless.

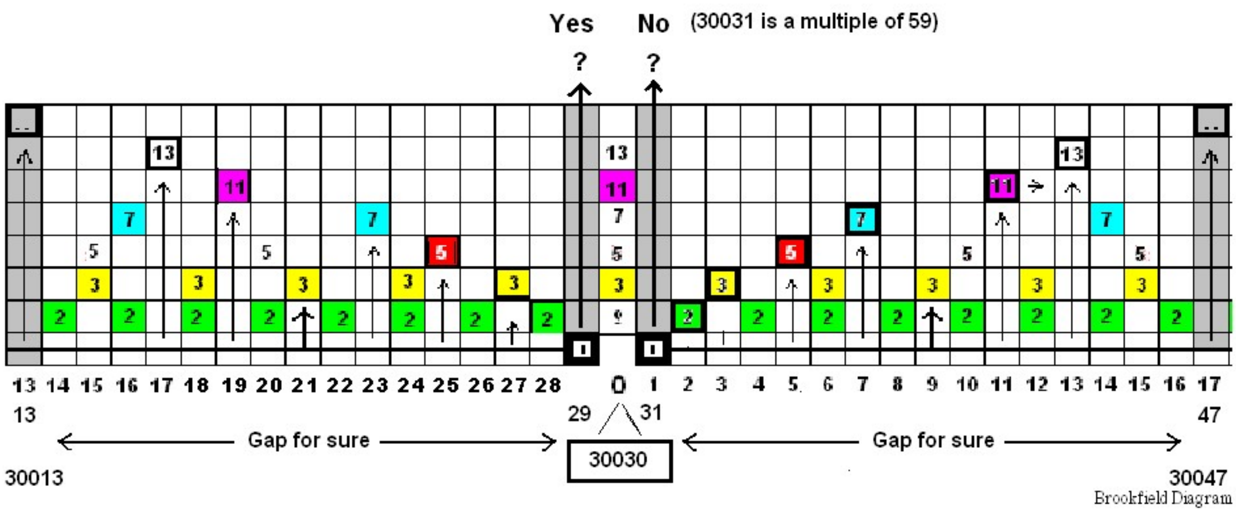


Fig.#4 *The completion of the* ***13 epicycle*** *(at 2x3x5x7x11x13) and the "inexplicable" gaps at 30,030 (and partition number divergence {Ivars Peterson 2000*}). Notice: the prime curve in reverse and then the restarting of the cycle at 30031. By definition an epicycle is a cycle and cycles are equally spaced by definition. Interference patterns are indeed patterns therefore not random or "inexplicable."*

It is worth noting that a gap of any given length can be found by merely multiplying primes together. A gap of at least 13 can be found at the completion of the 13 epicycle. A gap of at least 17 can be found at the completion of the 17 epicycle (*at 2x3x5x7x11x13x17****) and a gap of at least 30029 can be found at the completion of the 30029 epicycle etc.,***[3]

There being no possible source of up-down **a**symmetry (from either simple cycles or more complex epicycles) the **targeting** for Goldbach paring is necessarily assured. That is to say, **with the entire number system being exhaustively described as a set of cycles and epicycles there is no possible source of (Goldbach defeating) asymmetry anywhere within the whole number system**. *This argument takes the form of Ep(a) – Ep(b) ≡ Ep(n) -- except when n=0 and Equidistant spacing (E) does not apply because there is no pattern{p}.*

The Goldbach Comet:

"The Goldbach Comet," is related to the "prime curve" (Fig#2.) and is produced by mapping the even numbers on the X- axis and the sum/yield of Goldbach pairs accrued for each even number on the Y axis. Unlike the prime curve, the Comet's "tail/curve" spreads out as evens increase in size. This spreading is due to the fact that some evens are "High Yield Evens" (HYE's) due to their prime factorability/resonance with the prime field. Others are "Low Yield

Evens"(LYE's) due to their lack of resonance with any relevant primes factors other than "2" {they are instead coupled with slope-irrelevant higher prime factors [see below]}. Every even number is by definition factorable by two and thus aligned with the mirror image (and equidistant), 2-resonance in the prime-field. The result is that every even number is bound to accrue Goldbach pairs, if not from large prime resonances (2&3&5&7 etc) then from the bottom-line 2-resonance.

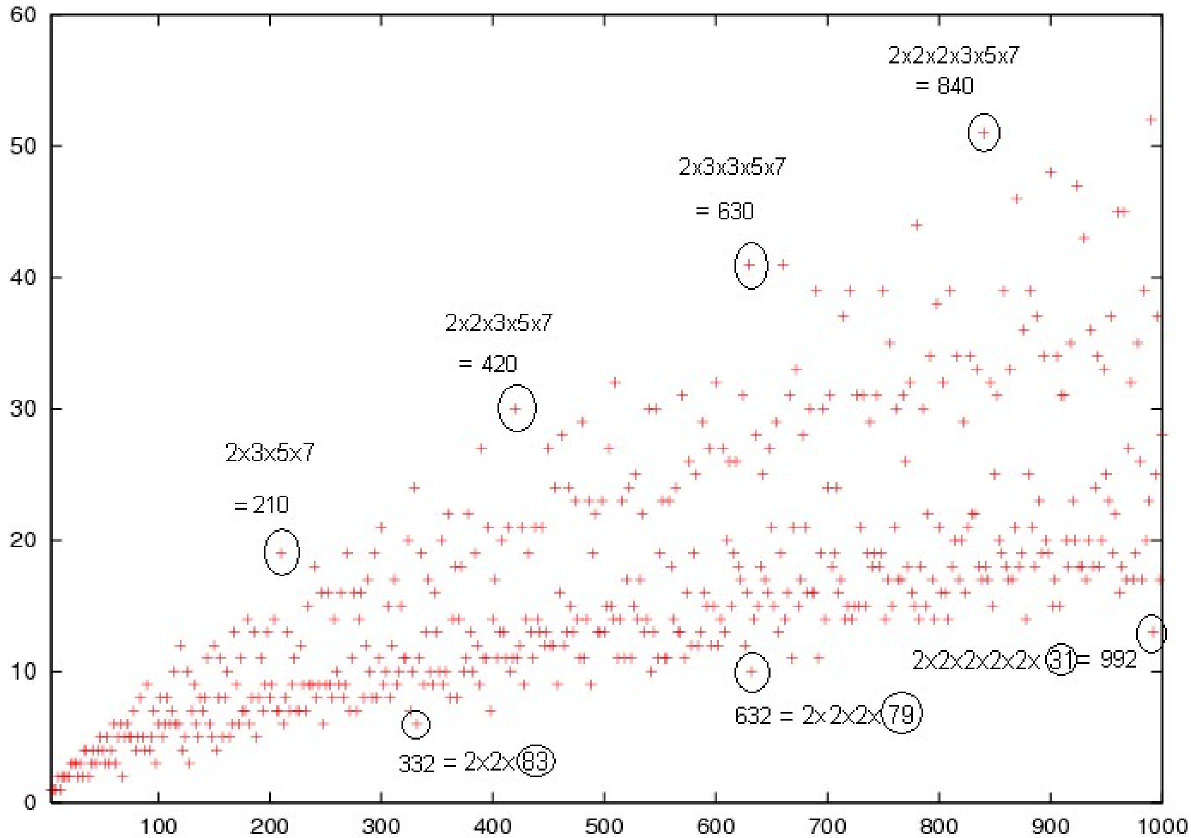


Figure #5. *"The Goldbach Comet" showing the upper and all-important lower threshold. If the lower threshold were to go to zero the conjecture would be defeated. The lower threshold however, cannot go to zero because all even numbers are, at very least, factorable by "2."*

The pair-limiting "factors" that produce the low threshold from the prime curve are as follows:

#1. The number "1" cannot be paired with 11 to produce 12 because 1 is not a prime. This means that for every *even number* a maximum of one pair might be missing (in the case that the lower adjacent number to the even is a prime. (Subtract one pair maximum per even)

#2. The even number in question might be the next multiple of a prime squared (12, 30,..992 etc.) I.E.,{3x3}+3=12. "12" cannot use 9+3 as pair. "14" cannot use "9" as well but "14" has more options. Numbers like 12, 30, 992 are on the "leading bleeding" edge of the prime curve. Subtract a maximum of one more pair (for a total of two).

#3. Epicycle Ending Gaps can appear to interfere (with pairing) but are proportional to the size of the last prime(linear) whereas the endings themselves are proportional to the prime multiplied by all previous primes (exponential). Epicycle endings are deceptive because these rare gaps (exponentially small) must be paid for by copious gap-less (equi-targeted) prime carpet (linear).

#4. The even number may be factorable only by 2, or by 2 and some slope-irrelevant higher prime and thus forced to occupy the 2-resonant lower band of the Goldbach Comet. "2-resonance," however, being established in eternity by the Prime "2" is eternally conserved and subsequently not a threat to the conjecture. On the contrary, it is the eternality of 2-resonance that verifies the targeting component of the conjecture.

The mere conservation of uniformity/equidistance/2-resonance however, does not, in and of itself, guarantee the validity of the Goldbach conjecture. It is entirely possible (and indeed certain) for the combined subtraction of the up-down symmetric odd numbers and the up-down symmetric evens to leave nothing at all. What is also needed is some comprehensive guarantee of prime **availability**.

Availability:

In order for primes to be coupled into Goldbach pairs, primes must be available. The slope of the "prime-curve" (Fig. #2) represents the density of primes at any given point with the 45 degree angle representing maximum density/availability (..2,3..) and zero slope representing zero density/availability(….). Were the slope to go to zero parallel to the x – axis there would be no more primes and the Goldbach conjecture would fail for any larger even numbers.

Luckily the slope can be analyzed and shown to conform to an eternal, square root function, as follows:

In order for a *prime multiple set* to be effective in changing the slope of the *prime curve (Fig.#2)* it has to be effective where smaller primes are not effective. If the smaller primes have already established the slope of the *prime-curve* then the larger prime is redundant and ineffective.

The question therefore is "*at what point does a prime multiple become non-redundant and thus effective in changing the slope of the prime curve*?" The answer is that prime multiples only become effective at the point of the prime squared. The reason for this is that a prime (say,7) is not effective at 7x2 because that is already covered by multiples of 2. Similarly "7" is not effective at 7x3 because that "re-sloping" has already been accomplished by multiples of "3." Moreover "7" is not effective at 7x4 because that "re-sloping" has already been accomplished by multiples of "2" etc Subsequently, the *largest effective conditioning prime* (for any given even number) is always given by the square root of that even number rounded down to the nearest prime. Thus, while odds (initial prime candidates[4]) are being produced at a linear rate (with the linear expansion of the system), candidate removal is being accomplished only at a square root rate.

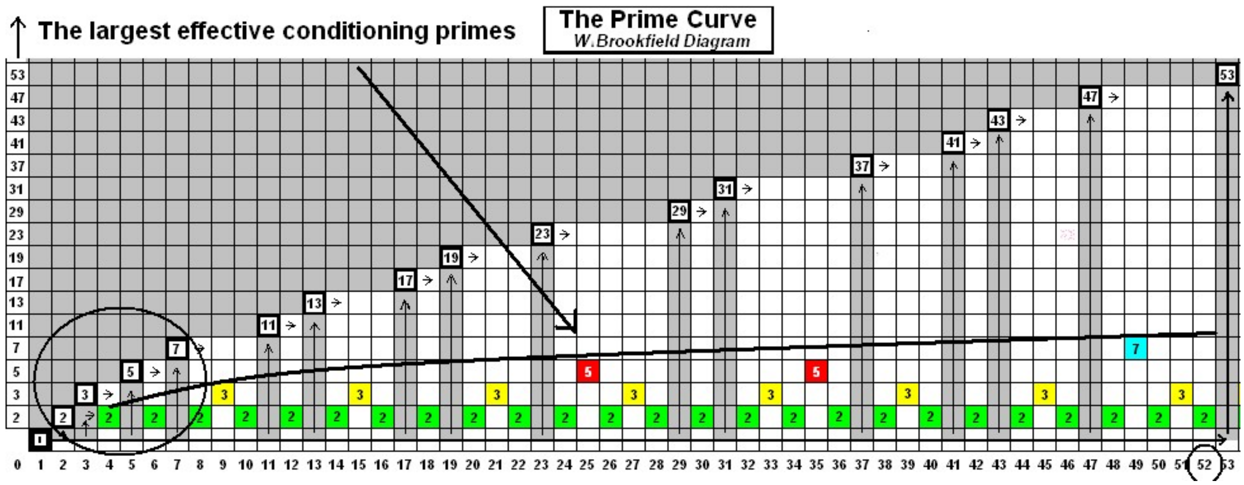


Fig #6. *The largest effective conditioning prime for 52 is 7. All larger primes are irrelevant.*

The result is that the number of primes **available** for pairing eternally increases (as even numbers get and larger). This is because the production (linear) of odds (initial prime candidates) eternally outpaces the *filtering out of odd prime candidates* (by a mere square root function of *effective prime-multiples)*.

The result is that both **Targeting** and **Availability** are assured for all **even numbers** greater than two and that *"every (finite) even integer greater than 2 can indeed be expressed as the sum of two primes."* This verifies the Goldbach conjecture for all finite evens.

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1. As a publicity stunt, the publishers of "Uncle Petros and the Goldbach Conjecture" (Bloomsbury USA in the U.S. and Faber and Faber in Britain) announced a $1 million prize for anybody who proved Goldbach's Conjecture within two years of the book's publication in 2000. The prize went unclaimed.

2. Just to avoid possible confusion here, prime #4 is seven. Prime # 5 is eleven.

3. The only condition under which smaller frequencies can be effectively blocked is when all primes work together to produce "end of epicycle gaps." Fig #4 .The size of these gaps however, grows only in a linear fashion while the distance from zero to gap increases exponentially.

4. Initially all numbers are "candidates for primehood" however the even prime number two immediately dispenses with the set of all evens higher than two – leaving only odd numbers as "prime candidates" for this analysis.

Addendum:

*While writing this article I came across an article by Ivars Peterson of the Mathematical Association of America. In his article he seems mystified by the large divergence in the number of Goldbach pairs between 30030 (905) and it's immediate even neighbors 30028(237) and 30032 (225). He seems unaware that 30030 = 2x3x5x7x11x13 and is highly "in phase," whereas 30028 and 30032 are far less in phase. 30034, being {2 x 15017} is particularly out of phase with the mirror-image prime-field.
"The (computer generated) data also show wide fluctuations in the number of partitions in going from one even integer to the next. For example, the number of partitions of 30,030 is 905, whereas the neighboring even integer 30,028 has 237 partitions and 30,032 has 225 partitions." -- Ivars Peterson's Math Trek – Goldbach Pairs - 2000